22PHP206

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2023
(Second Semester)

Branch – **PHYSICS**

MATHEMATICAL PHYSICS WITH NUMERICAL METHODS

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. The function $f(Z_o) = \frac{1}{2\pi i}\int \frac{f(Z)}{Z-Zo}\, dz$ is called ________.
   (i)Analytic function (ii)Cauchy's integral formula
   (iii)Cauchy Residue formula (iv)Laplace equation

2. The value of L(1) is ________.
   (i)$\frac{1}{s}$ (ii)$\frac{1}{s^2}$ (iii)Zero (iv) infinity

3. The value of P_o (x) is ________.
   (i)one (ii)Zero (iii)i (iv)-i

4. The three dimensional heat flow equation is ________.
   (i)$\nabla^2 U = \frac{1}{h^2}\frac{\partial U}{\partial t}$ (ii)$\nabla^2 U = \frac{1}{h}\frac{\partial U}{\partial t}$ (iii)$\nabla^2 U = \frac{1}{U}\frac{\partial U}{\partial t}$ (iv) $\nabla^2 U = \frac{\partial U}{\partial t}$

5. In bisection method, the approximate root Xo is ________.
   (i)$\frac{a-b}{2}$ (ii) $\frac{a+b}{2}$ (iii)$\frac{axb}{2}$ (iv)$\frac{a \div b}{2}$

SECTION - B (15 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. a. State and prove Cauchy's integral formula.
   OR
   b. Define singularity of an analytical function and briefly explain their types.

7. a. State and prove convolution theorem with respect to Fourier Transform.
   OR
   b. Explain the properties of Laplace transform.

8. a. Obtain the solution of Laplace equation in Cartesian coordinates using the method of separation of variables.
   OR
   b. Show that $J_{1/2}(x) = \sqrt{2/\pi x}\, \sin x$.

9. a. Obtain the D'Alembert's solution for the vibrating string.
   OR
   b. Explain the vibrations of a rectangular membrane.

Cont...

10. a. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by (i) Trapezoidal rule and by (ii) Simpson's one third rule.

OR

b. Fit a straight line to the data given below.

X	2	1	0	-1	-2
Y	9	7	5	3	1

Also estimate the value of y at x=2.5.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11. a. Define Cauchy-Riemann equation for an analytic function.

OR

b. State and prove the theorem connected with Taylor's series.

12. a. Explain the Fourier sine and cosine transforms of a derivative.

OR

b. Derive the Laplace transform of a periodic function and find $L(e^{at}\cos\omega t)$.

13. a. Obtain the Rodrigue's formula for Legendre polynomials.

OR

b. Obtain the generating function for Jn (x).

14. a. Determine the steady state temperature distribution in a thin plate bounded by the lines x=0, x=l, y=0 and y= ∞; assuming that the heat cannot escape from either surface of the plate, the edges x=0, x=l, y=∞ being kept at zero temperature, while the edge y=0 being kept at steady state temperature F(x). Use PDE to solve.

OR

b. Derive the equation motion for a vibrating string.

15. a. Write down the algorithm of IV order R-K method and apply it to find y(0.2) given that y^1=x+y, y(0) = 1.

OR

b. Explain iteration method of finding the real root and use the analogy to solve X^3=2X+5. Obtain positive root by iteration method.

Z-Z-Z END