20MAP08

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2023
(Second Semester)

Branch – **MATHEMATICS**

TOPOLOGY

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 In discrete topology on the set X, every set is ________.
i) Open ii) closed
iii) open and closed iv)either open or closed

2. If $d(x,y)=\|x-y\|=\{(x_1-y_1)^2+,\ldots\ldots\ldots\ldots+(x_n-y_n)^2\}^{1/2}$ is called _____ metric.
i) Discrete ii) Euclidean iii) Standard bounded iv) square

3. The ordered square I_0^2 is ____.
i) Connected ii) path
iii) connected but not path connected iv) connected but path connected

4. ____ is not locally compact.
i) R^n ii) real line R
iii) Q iv) compact space

5. ______________ is completely regular.
i) A subspace of a completely regular space ii) A product of completely regular space
iii) Every locally compact Hausdorff space iv) all

SECTION - B (15 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a) Let Y be a subspace of X. Let A be a subset of Y let $\overline{A}$ denote the closure of A in X. Then prove the closure of A in Y equals $\overline{A}\cap Y$

OR

b) If B is a basis for the topology of X and C is a basis for the topology of Y. then prove that the collection D= {BxC/ BεB and CεC} is a basis for the topology of $X\times Y$.

7 a) State and prove the Pasting lemma.

OR

b) Let d and d' be two metrics on the set X. Let τ and τ' be the topologies they induce respectively. Then prove that τ' is finer than τ iff for each x in X and each $\in>0$, there exists a $\delta>0$ such that $B_{d'}(x,\delta)\subset B_d(x,\in)$

8 a) Apply the basic results, prove that the image of a connected space under a continuous map is connected.

OR

b) Let $f:X\to Y$ be continuous map, where X is a connected space and Y is an ordered set in the order topology. If a and b are two points of X and if r is a point of Y lying between f(a) and f(b), then show that there exists a point c of X such that f(c)=r.

Cont...

9 a) Let X be locally compact Hausdorff let A be a subspace of X. If A is closed in X or open in X, then show that A is locally compact

OR

b) Suppose that X has a countable basis. Then prove that (i) every open covering of X contains a countable sub collection covering X (ii) There exists a countable subset of x that is dense in X.

10 a) Prove that every compact Hausdorff space is normal.

OR

b) Prove that every metrizable space is normal.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a) Let X be a topological space. Then prove that the following conditions hold:

(i) Φ and X are closed

(ii) Arbitrary intersections of closed sets are closed

(iii) Finite unions of closed sets are closed.

OR

b) Let B *and* B' be bases for topologies τ *and* τ' respectively on X. Then prove that the following are equivalent

(i) τ' is finer than τ

(ii) For each $x \in X$ and each basis element $B \in B$ containing x, there is a basis element $B' \in B$ such that $x \in B' \subset B$

12 a) Let X and Y be topological spaces; let $f : X \to Y$. Then prove that the following are equivalent:

(i) f is continuous

(ii) For every subset A *of* X, one has $f(\overline{A}) \subset \overline{f(A)}$

(iii) For every closed set, the set $f^{-1}(B)$ is closed in X.

(iv) For each $x \in X$ and each neighborhood V *of* $f(x)$, there is a neighborhood U *of* x such that $f(U) \subset V$

OR

b) State and prove sequence lemma.

13 a) Prove that If L is linear continuum in the order topology then L is connected, and also are intervals and rays in L.

OR

b) State and prove the Tube lemma.

14 a) Apply the basic results, Let X be a metrizable space. Then prove the following are equivalent:

(*i*) X is compact

(*ii*) X is limit point compact

(*iii*) X is sequentially compact

OR

b) Make use of the logical arguments, prove that

(i) A subspace of a Hausdorff space is Hausdorff, a product of Hausdorff is Hausdorff.

(ii) A subspace of a regular space is regular; a product of regular space is regular.

15 a) Analyze the statement of Tietze Extension theorem.

OR

b) Analyze the statement of Imbedding theorem.

Z-Z-Z END