22MAP207

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2023
(Second Semester)

Branch – **MATHEMATICS**

PARTIAL DIFFERENTIAL EQUATIONS

Time: 3 hours Maximum Marks: 50

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. The partial differential equation corresponding to the equation $(x-a)^2+(y-b)^2+z^2=1$ is ____
   (i) $z=(x+a)(y+b)$ (ii) $z^2(1+p^2+q^2)=1$
   (iii) $2z=(ax+y)^2+b$ (iv) $ax^2+by^2+z^2=1$

2. The partial differential equation of the form $Pp+Qq=R$, where P, Q and R are functions of x, y, z is called _______
   (i) Clairaut's equation (ii) Characteristic equation
   (iii) Charpit's equation (iv) Lagrange's equation

3. The second order partial differential equation is elliptic if ____
   (i) $S^2-4RT>0$ (ii) $S^2-4RT=0$
   (iii) $S^2-4RT<0$ (iv) $S^2-4RT\neq 0$

4. The displacement $\xi(x,t)$ in longitudinal vibration of a bar satisfies the wave equation $\frac{\partial^2\xi}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2\xi}{\partial t^2}$ where $c^2=$ ______.
   (i) $\frac{E}{\rho}$ (ii) $\frac{T}{\rho}$ (iii) $\frac{T}{\sigma}$ (iv) $\frac{1}{LC}$

5. The diffusion equation is represented by ______.
   (i) $u_{tt}-c^2\Delta^2u=0$ (ii) $u_t-k\Delta^2u=0$
   (iii) $\Delta^2u=0$ (iv) $\Delta^2u=f(x,y,z)$.

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6. (a) Find the general integral of the linear partial differential equation $y^2p-xyq=x(z-2y)$.

   OR

   (b) Find the surface which intersects the surfaces of the system $z(x+y)=c(3z+1)$ orthogonally and which passes through the circle $x^2+y^2=1, z=1$.

7. (a) If u is the complementary function and z_1 a particular integral of a linear partial differential equations, then show that $u+z_1$ is a general solution of the equation.

   OR

   (b) Find the particular integral of $(D^2-D')=2y-x^2$.

Cont...

8. (a) If $\rho > 0$ and $\Psi(r) = \int_V \frac{\rho(r')dr'}{|r-r'|}$, where the volume V is bounded, then prove that $\lim_{r\to\infty} r\,\Psi(r) = M$ where $M = \int_V \rho(r')dr'$.

OR

(b) Explain briefly about the two main types of the boundary value problem for that Laplace's equation.

9. (a) Write any two applications of wave equation in Physics.

OR

(b) Show that $y = f(x+ct) + g(x-ct)$ is the general solution of wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$ where f and g are arbitrary fuctions.

10. (a) Analyze the occurrence of diffusion equation in conducting media.

OR

(b) Show that $\theta = \frac{1}{\sqrt{t}} exp\left(-\frac{x^2}{4\kappa t}\right)$ is a solution of the diffusion equation $\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{\kappa}\frac{\partial \theta}{\partial t}$.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11. (a) Show that the equations $xp - yq = x;\ x^2p + q = xz$ are compatible and solve them.

OR

(b) Find the complete integral of the equation $p^2x + q^2y = z$.

12. (a) Derive the Laplace equation in cylindrical coordinates.

OR

(b) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and solve it.

13. (a) Show that the surfaces $x^2 + y^2 + z^2 = cx^{2/3}$ can form a family of equipotential surfaces and find the general form of the corresponding potential function.

OR

(b) Find uniform insulated sphere of dielectric constant κ and radius a carries on its surface a charge density $\lambda P_n \cos\theta$, show that the interior of the sphere contributes an amount $\frac{8\pi^2\lambda^2 a^3 \kappa n}{(2n+1)(\kappa n+n+1)^2}$ to the electrostatic energy.

14. (a) Derive the D'Alembert's solution of one dimensional wave equation.

OR

(b) Find approximate values for the first three eigenvalues of a square membrane of side 2.

15. (a) State and prove the Duhamel's theorem.

OR

(b) Show that the Poisson integral $\theta(x,t) = \frac{1}{2(\pi k t)^{1/2}} \int_{-\infty}^{\infty} \phi(\xi) e^{\frac{-(x-\xi)^2}{4kt}} d\xi$ is the solution of the initial value problem

$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{k}\frac{\partial \theta}{\partial t} - \infty < x < \infty, \theta(x,0) = 0;\ \theta(x,t) = \phi(x), t \to 0.$

Z-Z-Z END