PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2023
(Second Semester)

Branch – **MATHEMATICS**

COMPLEX ANALYSIS

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 The cross ratio is ------- under linear transformation.
(i) Analytic (ii) Invarient
(iii) Straight line (iv) Symmetric

2 ------- is called the conjugate differential of du.
(i) du (ii) du^*
(iii) u (iv) $*du$

3 A function which is analytic in the whole plane is said to be _____.
(i) Uniform (ii) Convergent
(iii) Entire (iv) Meromorphic

4 An analytic function g(Z) in Ω is univalent if $g(Z_1) = g(Z_2)$
(i) For any Z_1 and Z_2 (ii) For fixed Z_1 and Z_2
(iii) Only for $Z_1 = Z_2$ (iv) For $Z_1 \neq Z_2$

5 The sum of the residues of an elliptic function is _____.
(i) Finite (ii) Infinite
(iii) Does not exist (iv) Zero

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a Find the linear transformation which carrier 0,i,-i into 1,-1,0.

OR

b Prove that a function which is analytic and bounded in the whole plane must reduce to a constant.

7 a Find the poles and residue of cot Z.

OR

b If u_1 and u_2 are harmonic in Ω, prove that $\int_\gamma u_1^* du_2 - u_2^* du_1 = 0$ for every γ which is homologous to Zero in Ω.

8 a State and prove Hurwitz theorem.

OR

b State and prove Taylor series expansion theorem.

Cont...

9 a Let f be a topological mapping of the region Ω onto the region Ω'. If the sequence $\{Z_n(t)\} or \{Z(t)\}$ tends to the boundary Ω, prove that $|f(Z_n)\ or\ f(Z(t))|$ tends to the boundary of Ω'.

OR

b Explain the Schwarz-Christoffel formula.

10 a Prove that a non constant elliptic function has equally many poles as it has zeros.

OR

b Prove that $\mathcal{P}(Z)$ is an elliptic function.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Prove that the cross ratio (Z_1, Z_2, Z_3, Z_4) is real if and only if the four points lie in a circle or on a straight line.

OR

b State and prove cauchy's theorem for a rectangle.

12 a Compute $\int_0^{\pi} \log \sin\theta \, d\theta$

OR

b State and prove Poisson's formula.

13 a State and prove Laurent series expansion theorem.

OR

b State and prove Mittag-Leffler theorem.

14 a State and prove Riemann mapping theorem.

OR

b State and prove Harnack's principle.

15 a. Prove that any two bases of the same module are connected by a unimodular transformation.

OR

b. Derive the canonical product representation of $\sigma(Z)$.

Z-Z-Z END