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PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION JUNE 2014
(Sixth Semester)

Branch – **MATHEMATICS**

ALGEBRA - II

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 Is the matrix $\begin{pmatrix} 1 & 2+3i & -1 \\ 2-3i & \sqrt{5} & \sqrt{2}+i \\ -1 & \sqrt{2}-i & \frac{3}{2} \end{pmatrix}$ is Hermitian?

2 Define symmetric matrix and give an example.

3 Define vector space homomorphism.

4 In $F^{(3)}$ check whether (1,1,0) (3,1,3) and (5,3,3) are linearly independent.

5 If F is the field of real numbers find A(W) where W is spanned by (1,2,3) and (0,4,-1).

6 If V is a vector space over F, $u \in V$ & $\alpha \in F$ then prove that $\|\alpha u\| = |\alpha|\,\|u\|$.

7 Give an example to show that an element in A(V) is right invertible but is not invertible.

8 Define matrix of a transformation in a basis.

9 If $T \in A(V)$ us unitary S.T. $TT^* = I$.

10 Prove that the relation of similarity is an equivalence relation in A(V).

SECTION - B (25 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Let A&B be complex matrices such that the product AB is defined. Then prove that i) $\overline{AB} = \overline{A}\,\overline{B}$ ii) $(AB)^* = B^*A^*$ iii) $(\overline{A})^{-1} = (\overline{A^{-1}})$ iv) $(A^*)^{-1} = (A^{-1})^*$. If A is non-singular.

OR

b State and prove Cayley-Hamilton theorem.

12 a Prove that $F^{(n)}$ is isomorphic to $F^{(m)}$ if and only if n=m.

OR

b If $\vartheta_1, \vartheta_2, \ldots\ldots \vartheta_n$ is a basis of V and if $w_1, w_2 \ldots\ldots w_n$ is V are linearly independent over F, ten prove that $m \le n$.

13 a Prove that A(A(W)) = W.

OR

b If V is finite dimensional and $\vartheta \neq 0 \in V$, then prove that there is an element $f \in \hat{V}$ such that $f(\vartheta) \neq 0$.

14 a If V is finite dimensional over F, then prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0.

OR

b If $\lambda \in F$ is a characteristics root of $T \in A(V)$, then prove that for any polynomial $q(x) \in F[x]$, $q(\lambda)$ is a characteristics root of q(T).

Cont...

15 a If $T \in A(V)$ is such that $(\vartheta T, \theta)=0$ for all $\vartheta \in V$, then prove that $T=0$.

OR

b If N is normal and AN=NA, then prove that $AN^*=N^*A$ where A,N are linear transformations on V.

SECTION - C (30 Marks)

Answer any **THREE** Questions
ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Prove that the characteristics roots of a Hermitian matrix are all real.

17 If V is the internal direct sum of $U_1, U_2, \ldots U_n$ then prove that V is isomorphic to the external direct sum of $U_1, U_2, \ldots U_n$.

18 Let V be a finite-dimensional inner product space then prove that V has an orthonormal set as a basis.

19 If V is n-dimensional over F and if $T \in A(V)$ has the matrix $m_1(T)$ in the basis $\vartheta_1, \vartheta_2, \ldots \vartheta_n$ and the matrix $m_2(T)$ is the basis $w_1, w_2, \ldots w_n$ of V over F, then prove that there is an element $C \in F_n$ such that $m_2(T)=Cm_1(T)C^{-1}$. If S is the linear transformation of defined by $\vartheta_i S=W_i$ for $i=1,2,\ldots n$ then prove that C can be chosen to be $m_1(S)$.

20 If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

Z-Z-Z END