PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2017
(Fifth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

REAL ANALYSIS

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 Define a equivalence relation.

2 Define a metric space.

3 Define a compact space.

4 Define cantor set.

5 Define a Cauchy sequence.

6 Define a power series.

7 Define about bounded in continuity and compactness.

8 Define a local maximum.

9 Define a local maximum.

10 State the mean value theorem.

SECTION - B (25 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Every infinite subset of a countable set A is countable. Prove it.

OR

b Prove that every neighborhood is an open set.

12 a Prove that a compact subset of a metric space is closed.

OR

b Prove that if E is an infinite subset of a compact set K, then E has a limit point in K.

13 a The sub sequential limits of a sequence $\{P_n\}$ in a metric space X form a closed subset of X. Prove it.

OR

b Given $\sum a_n$, put $\alpha = \limsup\limits_{n\to\infty} \sqrt{|A_n|}$, prove the following

(i) if $\alpha < 1$, $\sum a_n$ convergent (ii) if $\alpha > 1$, $\sum a_n$ divergent
(iii) if $\alpha = 1$, test gives no information.

14 a Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Prove that f(x) is compact.

OR

b Suppose f is a continuous 1-1 mapping of a compact metric spaced X onto metric space Y. Then the inverse mapping f^1 defined on Y by $f^1(f(x)) = x$ is a continuous mapping of Y onto X . Prove it.

Cont ...

15 a Let f be defined on [a, b]. If f has a local maximum at a point $x \varepsilon (a, b)$ and $f'(x)$ exists then $f'(x) = 0$ prove it.

OR

b Suppose f is a real differential function on [a, b] and $f'(a) < \lambda < f'(b)$. Then there is a point $x \varepsilon (a, b)$ such that $f'(x) = \lambda$ prove it.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Prove that a set is open if and only if its complement is closed.

17 Show that every K-cell is compact.

18 (a) Let $\{P_n\}$ converges to p x if and only if any neighborhood of p contains p_n for all but infinitely many n prove it

(b) If $p \varepsilon x$, $p' \varepsilon x$ and if $\{p_n\}$ converges to p and p', Show that $p = p'$.

19 Let f be a continuous mapping of a compact metric space X into metric space Y. Show that f is uniformly continuous on X.

20 If f and g are continuous real functions on [a, b] which are differentiable in (a, b). Then there is a point $x \varepsilon f(a, b)$ at which $[f(b) - f(a)]g'(x) = [g(a)-g(b)]f'(x)$ prove it.

Z-Z-Z END