(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2017

(Fifth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

DISCRETE MATHEMATICS & GRAPH THEORY

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 Define connectives. Give three logical connectives.

2 Define the negative of a biconditional statement.

3 Define a partial order relation.

4 Define an invertible function.

5 Define a modular lattice.

6 Define a binary relation.

7 Draw a disconnected graph with three components.

8 Define a walk in a graph.

9 Define a enterian graph.

10 Prove that there is only one path between every pair of vertices in a tree.

SECTION - B (25 Marks)

Answer **ALL** Questions

ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a With the help of truth tables, prove that
1) $\sim(p \wedge q) = \sim p \vee \sim q$ 2) $p \oplus q = (p \wedge \sim q) \vee (\sim p \wedge q)$.

OR

b Show that the following statement are contingency.
1) $(p \wedge \sim q) \vee (\sim p \wedge q)$ 2) $\sim(p \vee q) \wedge (\sim p \vee \sim q)$.

12 a If R is the set of real numbers, then show that the function $f : R \to R$ defined by $f(x) = 5x^3 - 1$ is one-one, and onto function.

OR

b Show that the function f defined by $f : Q \to Q$ such that $f(x) = 3x + 4$, for all $x \in Q$ is one-one, and onto where Q is the set of all rational numbers. Also find a formula that defines the inverse function f^{-1}.

13 a Show that the lattice $L = \{1, 2, 3, 6\}$ under divisibility relation and the lattice $(P(S), \leq)$ where $S = \{a, b\}$ are isomorphic.

OR

b Let L be a complemented distributive lattice. Then prove that Demorgan's laws
1) $(a \vee b)' = a' \wedge b'$ 2) $(a \wedge b)' = a' \vee b'$.

14 a Define a walk, a path, a circuit and a component of a graph. Give example for each.

OR

b Prove that if a graph has exactly two vertices of odd degree, there must be a path joining these two vertices.

Cont ...

15 a Define union, intersection and ring sum of two graphs and give example for each one.

OR

b Prove that a graph G with n vertices, (n – 1) edges, and no circuit is connected.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 If p, q and r any three statements, then using the truth table, prove that

a) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

b) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

c) $(p \wedge q) \vee r \equiv (p \vee r) \wedge (q \vee r)$

17 i) The relation R on a set S of all real numbers is defined as aRb if and only if $1 + ab > 0$. Show that this relation is reflexive , symmetric but not transitive.

ii) Define a composition relation and power of a relation R. Give example for each.

18 i) Show that the dual of a lattice is a lattice.

ii) Prove that every finite lattice L is bounded.

19 i) Prove that the number of vertices of odd degree in a graph is always even.

ii) Define a simple graph & give an example .

iii) The sum of degrees of all vertices in a graph 'G' is twice of number edges in G.

20 Prove that a given connected graph G is an Euler graph if and only if all the vertices of G are of even degree.

Z-Z-Z END