

Branch – MATHEMATICS WITH COMPUTER APPLICATIONS

DIFFERENTIAL EQUATIONS LAPLACE TRANSFORMS & FOURIER SERIES

Time : Three Hours

Maximum : 75 Marks

SECTION-A (20 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (10 x 2 = 20)

- 1 Solve $y = (x - a)p - p^2$.
- 2 Find the complementary function of $(D^2 - 8D + 9)y = 8 \sin 5x$.
- 3 Eliminate the constants a and b from $z = ax + by + a$.
- 4 Solve the equation $pq = k$.
- 5 Prove that $L\{f(t) + \phi(t)\} = L\{f(t)\} + L\{\phi(t)\}$.
- 6 Find $L[at^2 + bt + c]$.
- 7 Find $L^{-1} \left[\frac{1}{(s+2)^2 + 16} \right]$.
- 8 Find $L^{-1} \left[\frac{1}{s(s+a)} \right]$.
- 9 Define Fourier series of a function.
- 10 Write the properties of odd function and even function in a Fourier series.

SECTION - B (25 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 5 = 25)

- 11 a Solve $p^2 + 2yp \cot x = y^2$.
OR
b Solve $(D^3 - D^2 - D + 1)y = 1 + x^2$.
- 12 a Eliminate the functions f and ϕ from the relation $z = f(x + ay) + \phi(x - ay)$.
OR
b Solve $p(1+q^2) = q(z-1)$.
- 13 a Find $L[\sin^2 2t]$.
OR
b Find the Laplace transform of $f(t) = e^{-t}$ when $0 < t < 4$;
 $= 0$ when $t > 4$.
- 14 a Find $L^{-1} \left[\log \frac{s+1}{s-1} \right]$.
OR
b Find $L^{-1} \left[\frac{1}{(s+1)(s^2 + 2s + 2)} \right]$.

Cont

- 15 a Express $f(x) = x$ ($-\pi < x < \pi$) as a Fourier series with the period 2π .

OR

- b If $f(x) = x$ when $0 < x < \frac{\pi}{2}$
 $= \pi - x$ when $x > \frac{\pi}{2}$,

then expand $f(x)$ as a sine series in the interval $(0, \pi)$.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks ($3 \times 10 = 30$)

16 Solve $x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} + y = \frac{1}{(1-x)^2}$.

- 17 i) Solve $p = y^2 q^2$
 ii) Solve $p + q = \sin x + \sin y$.

18 Find i) $L[t e^{-t} \sin t]$ ii) $L\left[\frac{\sin at}{1}\right]$.

19 Solve $\frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} - 5y = 5$, given that $y = 0$, $\frac{dy}{dt} = 2$ at $t = 0$.

- 20 Express $f(x) = \frac{1}{2}(\pi - x)$ in the interval $(0, 2\pi)$ as a Fourier series with the period 2π .

Z-Z-Z

END