PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2017
(Sixth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

COMPLEX ANALYSIS

Time : Three Hours   Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 State the Jordan curve theorem.
2 Define a Harmonic function.
3 Define Jacobian of a transformation.
4 Define conformal mapping.
5 Define the length of an ar C L.
6 State Cauchy Goursat theorem.
7 Define a removable singularity.
8 Define residue at a pole.
9 State Cauchy's residue theorem.
10 State Jordan's Lemma.

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a If f(z) = u + iv is an analytic function of z = x + i y and ψ any function of x and y with differential coefficient of the first and second orders, then prove that

$$\left(\frac{\delta\psi}{\delta x}\right)^2+\left(\frac{\delta\psi}{\delta y}\right)^2=\left\{\left(\frac{\delta\psi}{\delta u}\right)^2+\left(\frac{\delta\psi}{\delta \vartheta}\right)^2\right\}|f'(z)|^2.$$

OR

b Let f (z) = u + iv be an analytic function in a domain D. Then prove that f(z) is constant in D if any one of the following conditions holds:
(i) f^{l} (z) vanishes identically in D.
(ii) R (f(z)) = u = constant.

12 a What is the region of the w – plane into which the rectangular region in the z-plane bounded by the lines x = 0, y = 0, x = 1 and y = 2 is mapped under the transformation $\omega = z + (2 - i)$?

OR

b Let the rectangular region D in the Z – plane be bounded by x = o, y = 0, x = 2, y = 3 determine the region D' of the w – plane into which D is mapped under the transformation $\omega = \sqrt{2}\ e^{i\pi/4} z$.

13 a State and prove gauss's mean value theorem.

OR

b Let f(z) be continuous in a simply connected domain D and Let $\int_c f(z)\, dz = o$ where c any rectifiable closed Jordan curve in D, Then prove that f(z) is analytic in D.

Cont ...

14 a Show that the function e^z has an isolated essential singularity at $z = \infty$.

OR

b State and prove Schwarz Lemma.

15 a Prove the $\int_{-\infty}^{\infty} \frac{\sin x dx}{x^2 + 4x + 5} = \frac{-\pi \sin 2}{e}$

OR

b prove that $\int_{0}^{\infty} \frac{1 - \cos x}{x^2} dx = \frac{\pi}{2}$.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 $f(z) = P + iQ$ is analytic function of $Z = x + iy$ and $P - Q = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$, Find f(z) subject to the conditions $f(\frac{\pi}{2}) = 0$.

17 Let f(z) be an analytic function of z in a region D of the z-plane and let $f^1(z) \neq 0$ inside D. Then prove that the mapping $\omega = f(z)$ is conformal at the points of D.

18 Let f(2) be analytic in the region $|z| < p$ and let $z = re^{i\theta}$ be any point of this region. Then prove that $f(re^{i\theta}) = \frac{1}{2\pi} \int_0^{2\pi} \frac{(R^2 - r^2) f(Re^{i\phi})}{R^2 - 2Rr\cos(\theta - \phi) + r^2} . d\phi$ Where R is any number such that $o \angle R \angle p$.

19 State and prove Rouche's Theorem.

20 Prove that P $\int_{o}^{\infty} . \frac{x^4}{x^6 - 1} dx = \frac{\pi\sqrt{3}}{6}$.

Z-Z-Z END