

Branch – MATHEMATICS WITH COMPUTER APPLICATIONS

CALCULUS

Time : Three Hours

Maximum : 75 Marks

SECTION-A (20 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (10 x 2 = 20)

- 1 If  $u = f(x, y, z)$  and  $x, y, z$  are all functions of  $t$ , find  $\frac{du}{dt}$ .
- 2 Define a saddle point.
- 3 Write the parametric formula for the radius of curvature at a point of a curve.
- 4 Define involute of a curve.
- 5 If  $f(x)$  is an even function of  $x$ , then prove that  $\int_{-a}^{+a} f(x)dx = 2 \int_0^a f(x)dx$ .
- 6 Evaluate  $\int x \cos ax \, dx$ .
- 7 Define a double integral.
- 8 Evaluate  $\int_0^1 \int_0^1 \int_0^1 dx dy dz$ .
- 9 Define Beta function.
- 10 Find the value of  $\Gamma(1/2)$ .

SECTION - B (25 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 5 = 25)

- 11 a Find  $\frac{du}{dt}$ , where  $u = x^2 + y^2 + z^2$ ,  $x = e^t$ ,  $y = e^t \sin t$  and  $z = e^t \cos t$ .  
OR  
b If  $u = a^3x^2 + b^3y^2 + c^3z^2$  where  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ , find the minimum value of  $u$ .
- 12 a Find the envelope of the family of straight lines  $y + tx = 2at + at^3$ , the parameter being  $t$ .  
OR  
b Find the radius of curvature at the point  $(a/4, a/4)$  to the curve  $\sqrt{x} + \sqrt{y} = \sqrt{a}$ .
- 13 a Prove that  $\int_0^{\pi} \theta \sin^3 \theta d\theta = \frac{2\pi}{3}$ .  
OR  
b Evaluate  $\int e^{2x} \cos 3x \, dx$ .

Cont ...

14 a Evaluate  $\iint xy \, dx \, dy$  taken over the positive quadrant of the circle  $x^2 + y^2 = a^2$ .

OR

b Change the order of integration in the integral  $\int_0^a \int_{x^2/a}^{2a-x} xy \, dx \, dy$  and evaluate it.

15 a Evaluate  $\int_0^{\pi/2} \sqrt{\tan \theta} \, d\theta$ .

OR

b Express  $\int_0^1 x^m (1-x^n)^p \, dx$  in terms of Gamma functions, and evaluate the

integral  $\int_0^1 x^5 (1-x^3)^{10} \, dx$ .

### SECTION - C (30 Marks)

Answer any **THREE** Questions

**ALL** Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Discuss the maxima and minima of the function  $x^3 y^2 (6 - x - y)$ .

17 Show that in the parabola  $y^2 = 4ax$  at the point  $t$ ,  $P$  radius of curvature =  $-2a (1+t^2)^{3/2}$ ,  $x = 2a + 3at^2$ ,  $y = -2at^3$ . Deduce the equation of the evolute.

18 Find a reduction formula for  $\int_0^{\pi/2} \sin^m x \cos^n x \, dx$ . where  $m, n$  are positive integers.

19 Evaluate  $\int_0^a \int_0^{x+y} \int_0^{x+y+z} e^{x+y+z} \, dz \, dy \, dx$ .

20 Evaluate in terms of Gamma functions the internal  $\iiint x^p y^q z^r \, dx \, dy \, dz$  taken over the volume of the tetrahedron given by  $x \geq 0, y \geq 0, z \geq 0$  and  $x + y + z \leq 1$ .

Z-Z-Z

END