(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2017

(Fourth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ANALYTICAL GEOMETRY OF 3D AND VECTOR CALCULUS

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 Find the length of the perpendicular from the point $(2,3,4)$ to the plane $3x-6y+2z+11=0$.

2 Prove that the planes $2x-2y+z+3=0$ and $4x-4y+2z+5=0$ are parallel.

3 Find the value of k so that the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-5}{1}=\frac{z-6}{-5}$ may be perpendicular to each other.

4 Find the distance of the point $(3,4,5)$ from the point of intersection of $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{5}$ with the plane $x+y+z=2$.

5 Find the equation of the sphere which has the line joining the points (2,7,5) and (8,-5,1) as diameter.

6 Find the centre and radius of the sphere $2x^2+2y^2+2z^2-2x+2y-4z-5=0$.

7 A particle moves along a curve whose parametric equations are $x=e^{-t}$, $y=2\cos 3t$, $z=2\sin 3t$ where t is the time. Find velocity and acceleration at $t=0$.

8 Prove that the vector $2xye^{z}i+x^2e^{z}j+x^2ye^{z}k$ is a irrotational vector.

9 State Green's theorem.

10 By using Stoke's theorem prove that $\int r.dr = o$ where $r = xi+yj+zk$.

SECTION - B (25 Marks)

Answer **ALL** Questions

ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Find the equation of the plane passing through the points $(2,-5,-3)$, $(-2,-3,5)$ and $(5,3,-3)$.

OR

b Find the equation of the plane which passes through the point $(1,-2,1)$ and is perpendicular to each of the planes $3x+y+z-2=0$ and $x-2y+z+4=0$.

12 a Find the symmetrical form of the equations of the line of intersection of the planes $x+5y-z-7=0$, $2x-5y+32+1=0$.

OR

b Find the volume of the tetrahedron whose vertices are $(3,2,3)$, $(0,3,4)$, $(6,1,4)$, $(6,3,2)$.

13 a Find the centre and radius of the circle determined by the sphere $s=x^2+y^2+z^2+10y-4z-8=0$ and the plane $x+y+z-3=0$.

OR

b Find the equation of the sphere having the circle $x^2+y^2+z^2-2x+4y-6z+7=0$, $2x-y+2z=5$ for a great circle.

Cont ...

14 a If $\nabla \phi = 2xyz^3 i + x^2z^3 j + 3x^2yz^2 k$, than find ϕ if $\phi(1,-2,2) = 4$.

OR

b If ϕ is a scalar point function prove that $\nabla . \nabla \phi = \nabla^2 \phi$ and $\nabla x (\nabla \phi) = \bar{o}$.

15 a The acceleration of a moving particle at time t is given by $a = 9t^2 i - 24^{t2} j + 4 \sin tk$. If $r = 2i + j$ and $\frac{dr}{dt} = -i - 3k$ at t = o. Find r .

OR

b A vector field is given by $f = (x^2 - y^2 + x)\, i - (2xy + y)\, j$. show that F is irrotational. Find its scalar potential. Hence evaluate the line integral from $(1,2)$ to $(2,1)$.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Find the bisector of the acute angle between the planes $3x + 4y - 5z + 1 = 0$, $5x + 12y - 13z = 0$.

17 Find the symmetrical form, the equations of the orthogonal projections of the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-4}{4}$ on the plane $3x + 4y + 5z = 0$.

18 Show that the two circles $x^2+y^2+z^2-y + 2z = 0$, $x - y + z - 2 = 0$ $x^2+y^2+z^2+x - 3y + z - 5 = 0$, $2x - y + 4z - 1 = 0$ lie on the same sphere and find its equation.

19 Prove that curl (curlf) = grad div f - ∇^2 f and if f is rolenoidal prove that $\nabla x \nabla x \nabla x \nabla x f = \nabla^4 f$.

20 Verify gauss divergence theorem for $f = (x^2 - yz)i + (y^2 - xz)j + (z^2 - xy)\, k$ taken over the rectangular parallelepiped enclosed by $x = o$, $x = a$, $y = o$, $y = b$, $z = 0$ and $Z = c$.

Z-Z-Z END