(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2017
(Fifth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ABSTRACT ALGEBRA

Time : Three Hours   Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks   (10 x 2 = 20)

1 Define an abelian group.

2 Prove that the identity element in a group G is unique.

3 State first Sylow's theorem of a group.

4 Define an automorphism of a group.

5 What do you mean by a division ring?

6 When will you say an integral domain is of finite characteristic?

7 Define a maximal ideal of a ring.

8 Define a Euclidean ring.

9 What do you mean by a primitive polynomial?

10 State the Eisenstein criterion.

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a If H is a non-empty finite subset of a group G and H is closed under multiplication, prove that H is a subgroup of G.

OR

b Let H, K be two subgroups of a group G. Then prove that HK is a subgroup of G if and only if HK = KH.

12 a Prove that every group is isomorphic to a subgroup of A(S) for some appropriate S, where A (S) is the set of all bijections on S.

OR

b Prove that every permutation on a set is the product of its cycles.

13 a If R is a ring, prove that (i) a0 = a0 = 0 (ii) a(-b) = (-a)b = -(ab) for all a, b $\in$ R

OR

b If ϕ is a homomorphism of a ring R into a ring R' with kernel I(ϕ), then prove that (i) I(ϕ) is a subgroup of R under addition.
(ii) If a $\in$ I(ϕ) and r $\in$ R, then ar, ra are in I(ϕ).

14 a Let R be a Euclidean ring. Suppose that for a, b, c $\in$ R, a | bc, but (a, b) = 1. Prove that a | c.

OR

b If P is a prime number of the form 4n + 1, prove that there exists a solution for congruence $x^2 \equiv -1$ (mod p).

Cont ...

15 a State and prove the division algorithm on polynomial rings.

OR

b State and prove Gauss Lemma.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 i) Prove that a subgroup N of a group G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

ii) Prove that N is a normal subgroup of G if and only if $gNg^{-1} = N$ for every $g \in G$.

17 State and prove Cauchy's theorem for abelian groups.

18 Let R and R' be rings and ϕ a homomorphism of R onto R' with kernel U. Prove that

i) R' is isomorphic to R/U.

ii) Moreover there is a one – to – one correspondence between the set of ideals of R' and the set of ideals of R which contains U.

iii) This correspondence can be achieved by associating with an ideal W' in R, the ideal W in R defined by $W = \{x \in R | \phi(x) \in W'\}$.

19 Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Prove that R is a field.

20 If R is unique factorization domain and if p(x) is a primitive polynomial in R[x], prove that it can be factored in a unique way as the product of irreducible elements in R[x].

Z-Z-Z END