18MAP03

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2018
(First Semester)

Branch –**MATHEMATICS**

ORDINARY DIFFERENTIAL EQUATIONS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

Choose the correct answer :

1 The x''-2tx + 2 nx is a Hermite equation of order ____________.
(i) t (ii) 2t (iii) 2n (iv) n

2 The generating function of the Bessel function $J_n(t)$ is ________.
(i) $\exp\left[\frac{2}{t}\left(x+\frac{1}{x}\right)\right]$ (ii) $\exp\left[\frac{t}{2}\left(x-\frac{1}{x}\right)\right]$
(iii) $\exp\left[\frac{1}{2}\left(x-\frac{1}{x}\right)\right]$ (iv) $\exp\left[\frac{t}{2}\left(x+\frac{1}{x}\right)\right]$

3 The equation $\bar{x}' = A(t)\bar{x} + \bar{b}(t), t \in I$ represents ________.
(i) a vector matrix representation of a linear non-homogenous system
(ii) a vector matrix representation of a linear homogeneous system
(iii) a vector matrix representation of a non-linear non-homogenous system
(iv) a vector matrix representation of a non linear homogenous system

4 The solution matrix of the system $\bar{x}' = A(t)\bar{x}$ is a fundamental matrix if
(i) the rows are linearly independent (ii) the columns are linearly independent
(iii) both the rows and the columns are linearly independent
(iv) neither rows and nor columns are linearly independent

5 For a constant matrix A, exp(tA), is
(i) $E + \sum_{p=0}^{\infty} \frac{t^p A^p}{p!}$ (ii) $E + \sum_{p=1}^{\infty} \frac{t^p A^p}{p!}$
(iii) $\sum_{p=0}^{\infty} \frac{t^p A^p}{p!}$ (iv) $E + \sum_{p=0}^{\infty} \frac{t^p A^p}{p}$

6 The necessary and sufficient condition for the linear system $x' = Ax$ to admit a non-zero periodic solution of period ω is that _______.
(i) E - $e^{A\omega}$ is singular (ii) E - $e^{A\omega}$ is non singular
(iii) E + $e^{A\omega}$ is non singular (iv) E + $e^{A\omega}$ is singular

7 The first approximation of x(t) of the IVP $x' = x^2$, x(0) = 1 is _______.
(i) 1-t (ii) 1+t (iii) 1 (iv) -1

8 The constants L, K for the IVP $x' = -x, x(0) = 1, t \geq 0, R = \{(t,x) : |t| \leq 1, |x-1| \leq 1,\}$ is
(i) K = 1 ; L =2 (ii) K = 2; L = 1
(iii) K = 1; L = ½ (iv) K = 1 ; L = 1

9 The equation $x'' + x = \sin 2t, x(0) = x(\pi) = 0$ is
(i) linear homogenous (ii) linear non homogenous
(iii) nonlinear homogenous (iv) nonlinear non homogeneous

10 The solution of the equation $x'' = f(t), x(0) = 0 = x(1)$ is x(t) = _____.
(i) $\int^{1} G(t,s) f(s) ds$ (ii) $\int^{1} G(t,s) ds$

SECTION - B (25 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Find a power series solution to the equation of motion of a simple pendulum $x''(t) + k\sin x(t) = 0$, where k is a constant with $x(0) = \frac{\pi}{6}$ and $x'(0) = 0$.

OR

b If P_n is a Legender polynomial, then show that $\int_{-1}^{1} P_n^{\,2}(t)dt = \frac{2}{2n+1}$.

12 a Express the system of three equations given by

$$(x_1, x_2 x_3)' = (4x_1 - x_2, 3x_1 - x_3 - x_{2,} x_1 + x_3)$$

in the vector-matrix form and hence solve the solution of the system.

OR

b If $\Phi(t), t \in I$, is a fundamental matrix of the system $\bar{x}' = A(t)\bar{x}$ such that $\Phi(0) = E$, where A is a constant matrix and E denotes the identity matrix. The show that Φ satisfies $\Phi(t+s) = \Phi(t)\Phi(s)$ for all values of t and s $\in I$.

13 a Find e^{At} when A = $\begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}$.

OR

b State and prove the Floquet theorem.

14 a State and prove the Gronwall inequality.

OR

b Solve the IVP $x' = x, x(0) = 1, t \geq 0$ by the method of successive approximations.

15 a Assume that (i) A, B are finite real numbers ; (ii) the functions $p'(t).q(t)$ and r(t) are real valued continuous functions on [A, B]; and (iii) m_1, m_2, m_3 and m_4 are real numbers. Also suppose that r(t) is positive on [A, B} or r(t) is negative on [A, B]. Then prove that all the eigen values of the Sturm – Liouville boundary value problem are real.

OR

b Construct the Green's function for L(x)=0 on (a, b) with the boundary conditions : $m_1x(a)+m_2x'(a) = 0$; $m_3x(b) + m_4x'(b)=0$.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a Construct the solution on (1, 1) of the equation $(1-t^2)x'' - 2tx' + 20x = 0; \quad x(0) = 1, x'(0) = 0$.

OR

b If a_1, a_2, a_3,... are the positive zeros of the Bessel function $J_p(t)$, then prove that $\int_0^1 fJ_p(a_m t)J_p(a_n t)dt = \begin{cases} 0 & if \ m \neq n \\ \frac{1}{2} j^2{}_{p+1}(a_n) & if \ m = n \end{cases}$.

17 Cont...

b Let A(t) be an n x n matrix which is continuous on . Suppose a matrix Φ satisfies $X' = A(t)X, t \in I$, then proves that det Φ satisfies the first order equation $(\det \Phi)' = (trA)(\det \Phi)$ or, in other words, for $\tau \in I, \det \Phi(t) = \det \Phi(\tau) \exp \int_{\tau}^{t} trA(s)ds$.

18 a Formulate the fundamental matrix for the system $\bar{x}' = A\bar{x}$, where $A = \begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}$.

OR

b Formulate the fundamental matrix of the linear system $\bar{x}' = A\bar{x}$, where $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{pmatrix}$.

19 a State and prove the Picard's thermo for the existence of a unique solution for a class of non-linear initial value problems.

OR

b Let x(t)=x(t; t_0, x_0) and x*(t)=x*(t, t_0*,x_0*) be solutions of the IVPs $x' = f(t,x)$, x(t_0)=x_0 and $x' = f(t,x)$, x*(t_0*)=x_0* respectively on an interval $a \le t \le b$. Further, let f $\in Lip(D,K)$ be bounded by L in D. Then prove that for any $\varepsilon > 0$, there exists $a\delta = \delta(\varepsilon) > 0$ such that $|x(t) - x^*(t)| < \varepsilon, a \le t \le b$ whenever $|t - t_0^*| < \delta$ and $|x - x_0^*| < \delta$.

20 a Using the method of separation of variables, solve the BVP :

$$\frac{\partial u}{\partial t}(x,t) = k\frac{\partial^2 u}{\partial x^2}(x,t), 0 < x < c, t > 0$$

$$\frac{\partial u}{\partial t}(0,t) = 0 \frac{\partial u}{\partial x}(c,t) = 0, t > 0$$

and u(x,0)=f(x),0 < x < c

OR

b Assume that the function f(t,x) in $x'' + f(t,x) = 0, x(a) = x(b) = 0, a \le t \le b$ --(1) satisfies the Lipschitz condition $|f(t,x) - f(t,y)| < K|x - y)|$ uniformly in t where K is a Lipschitz constant such that $p = \frac{K(b-a)^2}{8} < 1$. Then prove that the sequence $x_n(t) = \int_a^b G(t,s) f(s, x_{(n-1)}(s))ds, n$ = 1, 2, 3,...converge to a function x which is the unique solution of (1) and in addition, an upper bound on the error is $\|x_n - x\| < \frac{p^n}{1-p}\|x_n - x_0\|$.

Z-Z-Z END