14MAP11

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2018
(Third Semester)

Branch –**MATHEMATICS**

CORE ELECTIVE-I - STOCHASTIC DIFFERENTIAL EQUATION

Time: Three Hours Maximum: 75 Marks

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 15 = 75)

1 a Explain the following stochastic models :
(i) Population Model (ii) Optimal portfolio problem.

b Prove that Brownian motion B_t is a Gaussian process.

OR

c Define Brownian motion and show that it has independent increments.

d State and prove Borel Cantelli lemma.

2 a Let $f \in v(0,T)$, then prove that there exists a t-continuous version of $\int_0^1 f(s,w)dB_s(w);\quad 0 \le t \le T$.

b State and prove Ito isometry.

OR

c Prove that $\int_0^1 s dB_s = tB_t - \int_0^1 B_s ds$.

d Prove that $M_t = B_t^2 - t$ is an F_t – Martingale.

3 a State and prove 1 – dimensional Ito formula.

OR

b State and prove the existence and uniqueness theorem for stochastic differential equation.

4 a State and prove 1-dimensional Kalman – Bucy filter theorem.

OR

b Solve the stochastic differential equation :
$dX_t = 0, X_t = X_0, E[X_0] = \overline{X}_0 = 0, E[X_0{}^2] = a^2$ with the observation
$dZ_t = X_t dt + m dV_1;\ Z_0 = 0$

5 a State and prove strong Markov property for Ito diffusion.

OR

b State and prove Dynkin's formula with proving necessary lemma.

s Z-Z-Z END