14MAP05

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
MSc DEGREE EXAMINATION DECEMBER 2018
(Second Semester)

Branch –**MATHEMATICS**

COMPLEX ANALYSIS

Time: Three Hours Maximum: 75 Marks

Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 15 = 75)

1 a If $T_1 z = \frac{z+2}{z+3}$, $T_2 z = \frac{z}{z+1}$ then find $T_1 T_2 z, T_2 T_1 z_1$, and $T_1^{-1} T_2 z$. (7)

b State and prove Cauchy's theorem in a disk. (8)

OR

c State and prove Cauchy's integral formula. (8)

d State and prove Morera's theorem. (7)

2 a State and prove argument principle. (10)

b If u_1 and u_2 are harmonic in Ω, then show that $\int_\gamma u_1^* du_2 - u_2^* du_1 = 0$ for every cycle γ which is homologous to omod Ω. (5)

OR

c Derive Poisson integral formula. Express Poisson integral formula into polar co-ordinates. (15)

3 a Let f(z) be an analytic function whose region of definition contains an annulus $R_1 < |z-a| < R_2$ show that f(z) can be developed in a general power series of the form $f(z) = \sum_{n=-\infty}^{\infty} A_n (z-a)^n$ and prove that the Laurent development is unique. (15)

OR

b Define infinte product. Show that the necessary and sufficient condition for the absolute convergence of the product $\pi_1^{\infty}(1+a_n)$ is the convergence of the series $\sum_1^{\infty} |a_n|$. (8)

c State and prove Poisson-Jenson's formula. (7)

4 a State and prove Riemann mapping theorem. (15)

OR

b State and prove Harnack's principle. (7)

c Prove that the continuous function u(z) which satisfies mean value property is Harmonic. (8)

5 a Prove that any two bases of the same module are connected by a unimodular transformation. (7)