18MAP01

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2018
(First Semester)

Branch **-MATHEMATICS**

ALGEBRA

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks!
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 The number of conjugate classes in S4 is
(i) 3 (ii) 5
(iii) 7 (iv) 11

2 The number of 13-Sylow subgroups of a group G of order $11^2 13^2$ is
(i) 1 (ii) 3
(iii) 5 (iv) 7

3 The remainder of dividing $f(x) = 3x^4 + x^3 + 2x^2 + 1$ by $g(x) = x^2 + 4x + 2$, where f(x) and g(x) belong to $z_5[x]$ is
(i) 4x + 1 (ii) 2x+3
(iii) x+4 (iv) 2x+l

4 The polynomial $x^2 - 2$ is
(i) reducible over Z (ii) reducible over Q
(iii) irreducible over Q (iv) irreducible over R

5 The degree of V2 over Q is
(i) 0 (ii) 1 (iii) 2 (iv) 3

6 The splitting field for $f(x) = x^4 - 5x^2 + 6$ e Q[x] is
(i) Q (V2) (ii) Q(V2,V3)
(w) Q(S) (iv)Q(V2,V5)

7 If f(x) = F(x) is irreducible and if the characteristic of F is 0, then f(x) has
(i) no roots (ii) no multiple roots
(iii) multiple roots (iv) distinct roots

8 If ‘G is a group of automorphisms of K, then the fixed field of G is the set of all elements *a e* k such that
(i) cr (a) = a, for some cr e G (ii) *σ*(a) * a, for some *a* e G
(iii) <*J*(a) = a, for all cr e G (iv) cr (a) * a, for all cr e G

9 The linear transformation *T* e *A(V)* is said to be unitary if for all u, v e *V*
(i) (vT, v) = 0 (ii) (uT, vT) = (u, v)
(iii) (uT, v) = (u, vT*) (iv) TT* = T*T

10 If T is Hermitian, then all its characteristic roots are
(i) o (ii) real (iii) complex (iv) 1

SECTION - B (25 Marks!
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5x5 = 25)

1 1 a If G is a finite group, then prove that $C_a = o(G)/o(N(a))$.
OR
b State and prove third part of Sylow’ theorem.

13 a If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.

OR

b Prove that r' defines an isomorphism of F(x) onto F'(l) with the property that $ar'' = a'$ for evry a e F.

14 a Prove that the polynomial f(x) € $F(x)$ has a multiple root if and only if f(x) and $f'(x)$ have a nontrivial common factor.

OR

b If K is a finite extension of F, then prove that G(K, F) is a finite group and its order, 0(G(K, F)) satisfies 0(G(K, F)) ^[K : F].

15 a Prove that the linear transformation T on V is unitary if and only is it takes an orthonormal basis of V into an orthonormal basis of V.

OR

b If T G $A(V)$, then prove the following :
(i) T^* e $A(V)$ (ii) (T*)* = T (iii) (S + T)* = S* + T*

SECTION -C (40 Marks!

Answer **ALL** questions

ALL questions carry **EQUAL** Marks ($5 \times 8 = 40$)

16 a If O(G) = p^2 where p is a prime number, then prove that 'G' is abelian,
b State and prove Cauchy's theorem.

OR

c State and prove Sylow's theorem.

17 a Define the content of the polynomial,
b State and prove the Einsteins criterion.

OR

c Define a unique factorization domain.
d If R is a unique factorization domain and if p(x) is a primitive polynomial in R(x), then prove that is can be factored in a unique way as the product of irreducible elements in R(x).

18 a Prove that the element a e K is algebraic over F is and only if F(a) is a finite extension of 'F\

OR

b State the remainder theorem.
c Prove that a polynomial of degree 'n' over a field can have at most 'n' roots in any extension field.

19 a Define a simple extension.

b If 'F' is of characteristic 'O' and of a, b are algebraic over F, prove that there exists an element c e $F(a,b)$ such that F(a, b) = F(c).

OR

c Prove that K is normal extension of F if and only if K is the splitting field of some polynomial over F.

20 a If T e $A(V)$ has all its characteristic roots in F, prove that there is a basis of 'V' in which the matrix of T is triangular.

OR

b If F is a field of characteristic 0, and if T e $AJV)$ is such that trV - 0 for all i > l, then prove that T is nilpotent.