PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2018
(First Semester)

Branch – **STATISTICS**

REAL ANALYSIS & MATRIX ALGEBRA

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 Let f be a derivable function at a point c $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$, find f from the following
(i) discontinuous at c (ii) discontinuous at x
(iii) continuous at c (iv) continuous at x

2 The function $f(x) = |x + 2$ is not differentiable at a point at x = 0.
(i) x = 2 (ii) x = -2
(iii) x = -1 (iv) x = 1

3 Indicate the maximum value of f(x, y), if r = $\frac{\partial^2 f}{\partial x^2}$, t = $\frac{\partial^2 f}{\partial y^2}$, d = $\frac{\partial^2 f}{\partial x \partial y}$.
(i) $rt - s^2 > 0$ and $r < 0$ (ii) $rt - s^2 > 0$ and $r > 0$
(iii) $rt - s^2 < 0$ and $r < 0$ (iv) $rt - s^2 < 0$ and $r > 0$

4 If $\lim_{x \to a} f(x) = 1$ and $\lim_{x \to a} g(x) = m$, then $\lim_{x \to a} \frac{f(x)}{g(x)} - \frac{1}{m}$ if
(i) m = 0 (ii) $m \neq 0$
(iii) m = 1 (iv) $m \neq 1$

5 If f be a bounded function defined on [a, b] and P_1 and P_2 two partitions of [a, b] such that P_2 is refinement of P_1 then
(i) $L(P_2, f) \leq L(P_1, f)$ (ii) $L(P_2, f) \leq U(P_1, f)$
(iii) $U(P_2, f) \leq U(P_1, f)$ (iv) $U(P_2, f) \leq L(P_2, f)$

6 Find L(P, f) and U (P, f), if f(x) = $x \forall x \in [0,3], P = \{0,1,2,3\}$ be a partition of P.
(i) 3, 6 (ii) 2, 8
(iii) 8, 2 (iv) 6, 3

7 Let M and m to be the upper and lower bounds of f(x) in [a, b] choose the correct answer.
(i) $m \leq f(x) \leq M$ (ii) $m > f(x) > M$
(iii) $m < f(x) > M$ (iv) $M \geq f(x) \geq m$

8 Let I = [a, b] be a closed and bounded interval then a finite set of points P = {x_0, x_1, ….x_n} such that a < x_0 < x_1 <….< x_n, is called
(i) partition of the interval (ii) segment of the partition
(iii) norm of the partition (iv) refinement of the partition

9 Mention $\rho(A)$, if $A = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$,
(i) 0 (ii) 1

Cont...

SECTION - B (25 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Discuss the nature of continuity and derivability of the function

$$f(x) = x\frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} \text{ if } x \neq 0 \text{ and}$$

f(0) = 1 is continuous at x = 0.

OR

b Prove that every derivable function is continuous.

12 a Describe the Limit, Continuity and Derivability of the functions of two variables.

OR

b Examine the limit and continuity of the function $f(x,y) = \begin{cases} \frac{x^3 + y^3}{x - y} & ;(x,y) \neq 0 \\ 0 & ;(x,y) = 0 \end{cases}$.

13 a If f : [a, b] →R is a bounded function and $P \in P[a,b]$, show that m (b-a)m(b-a) $\leq L(P,f) \leq U(P,f) \leq M(b-a)$.

OR

b If f: [a, b] →R is a bounded function show that $\underline{\int_a^b} f(x)dx \leq \overline{\int_a^b} f(x)$.

14 a Explain Riemann – Stieltjes integral and its properties of upper and lower sums.

OR

b The function f(x) = x, $\alpha(x)$ = x². Does $\int_0^1 f \, d\alpha$ exists.

15 a Calculate the rank of the matrix A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$.

OR

b Describe Hermitian form and Quadratic form.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a If f and g are two continuous function at , prove that the function max (f, g) and min (f, g) are both continuous at a.

OR

b If f and g be a two function defined on [a, b] and derivable at any point a then $c \in [a,b]$, then prove that

(i) $(f+g)'c = f'(c) + g'(c)$ (ii) $(fg)'(c) = f(c)g'(c) + g(c)f'(c)$

(iii) $\left(\frac{f}{g}\right)'(c) = \frac{g(c)f'(c) + f(c)g'(c)}{[g(c)]^2}$

17 a State and prove a necessary and sufficient condition for the maximum and minimum and also the existence of extreme values.

OR

b Examine the points where a function $f(x,y) = xy + \frac{1}{x} + \frac{1}{y}; x > 0, y > 0$ has a

18 a State and prove Durabox theorem.

OR

b If $f \in R[a,b]$, prove that (i) $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$ if $b \geq a$,

(ii) $m(b-a) \geq \int_a^b f(x)dx \geq M(b-a)$ if $b < a$.

19 a State and prove the necessary and sufficient condition for RS. Integrals.

OR

b If $f \in R(\alpha_1)$ and $f \in R(\alpha_2)$, then prove that

$$\int_a^b f \, d(\alpha_1 + \alpha_2) = \int_a^b f \, d(\alpha_1) + \int_a^b f \, d(\alpha_2).$$

20 a Prove the Cayley – Hamilton theorem that every matrix satisfies its characteristic function.

OR

b Explain G inverse and their properties.

Z-Z-Z END