14MCU24

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2017
(Sixth Semester)

Branch MATHEMATICS WITH COMPUTER APPLICATIONS

LINEAR ALGEBRA

Time: Three Hours Maximum: 75 Marks

SECTION-A (20 Marks)
Answer ALL questions
’ > ALL questions carry EQUAL marks (10x2 = 20)

1 Define scalar and unit matrices.

2 Find the adjoint of A = 3 2 ^ 1 -1

3 Define vector space.

4 Define linear span.

5 Define algebraic extension.

6 Define splitting field. - ' '

7 When T e A (V) is said to be singular and regular.

8 Define characteristic roots of a matrix.

9 Define unitary transformation.

10 Define index of nilpotence.

SECTION - B (25 Marks)
Answer ALL Questions
ALL Questions Carry EQUAL Marks (5x5 = 25)

11 a Show that A = cos 9 sinG -sin0 cos0 is orthogonal and find the inverse of A.

OR

b Define the following with examples
(l)Hermitian matrix 4 (2) Skew Hermitian.

12 a State and prove Schwarz inequality.

OR

b Prove that L(S) is a subspace of V.

13 a If L is an algebraic extension of K and if K is an algebraic extension of F, then L is an algebraic extension of F. *

OR

b State and prove the Remainder theorem.

14 a If V is finite - dimensional over F, then prove that T e A(V) is invertible if and only if the constant term of the minimal polynomial for T is not 0.

OR

b The element *X* e F is a characteristic root of T *e* A(V) if and only if for some v * 0 in V, vT = *Xv*.

15 a If u G V! is such that uT"1^ = 0, where 0 < k ^ n_{i5} then prove that u = u_0T^k for some Uo € Vi>

OR , '

« b If N is normal and AN = NA, then prove that AN - N*A.

SECTION - C (30 Marks)

Answer any THREE Questions

ALL Questions Carry EQUAL Marks (3 x 10 = 30)

16 Find the characteristic roots and characteristic vectors of $\begin{vmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{vmatrix}$

17 State and prove the Gram-Schmidt Orthogonalization process.

18 If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F. Moreover, [L : F] = [L : K] [K : F].

19 If *X e* F is a characteristic root of T e A(V), then *X* is a root of the minimal polynomial of T. In particular, T only has a finite number of characteristic roots in F.

20 If V is n-dimensional over F and if T *e* A(V) has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F.

Z-Z-Z END