

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION MAY 2017
(Sixth Semester)

Branch - MATHEMATICS WITH COMPUTER APPLICATIONS

COMPLEX ANALYSIS

Time : Three Hours

Maximum : 75 Marks

SECTION-A (20 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (10 x 2 = 20)

- 1 Define analytic function.
- 2 Write down the Cauchy - Riemann equations in polar form.
- 3 Define conformal mapping.
- 4 Explain the transformation $w = c+z$ v
- 5 Define simply connected region.
- 6 Evaluate $\int_0^1 j(x^2 - iy)dz$ along $y = x$.
- 7 What are the poles of $f(z) = \frac{1}{(z-5)^3(z-4)^2}$?
- 8 State maximum modulus principle.
- 9 State Cauchy Residue theorem.
- 10 Find the residue of $f(z) = \frac{z^3}{(z-1)(z-2)(z-3)}$ at the pole $z = 2$.

SECTION - B (25 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 5 = 25)

- 11 a Show that the function $f(z) = z$ is nowhere differentiable.
OR
b Determine where $f(z)$ exists and find its value when $f(z) = x^2 + iy^2$.
- 12 a Find the image of the circle $|z| = 2$ by the transformation $w = z + 3 + 2i$.
OR
b Explain the transformation $w = \frac{1}{z}$.
- '13 a Evaluate $\int_C jz^2 dz$, where the ends of C are A(1, 1) and B(2, 4) given that
C is the curve $y = x^2$.
OR
b Evaluate $\int_C j \log z dz$, where C is the unit circle $|z| = 1$.

14 a Obtain Taylor's series to represent the function $\frac{-1}{(z+2)(z+3)}$ in the region $|z| < 2$.

OR

b State and prove Cauchy inequality.

15 a Find the residue of $f(z) = \frac{1}{(z-2)(z+1)^2}$ about each singularity.

OR

b Evaluate $\int_C \frac{z-1}{z(z+1)(z-3)} dz$, where C is $|z| = 2$.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Find an analytic function whose imaginary part is $3x^2 - y^2$.

17 Find the image of the circle $|z - 1| = 1$ in the complex plane under the image $\frac{1}{z}$.

18 State and prove Cauchy's Integral formula.

19 State and prove Liouville's theorem.

20 Show that $f(z) = \frac{1}{a + b \cos \theta} = \frac{1}{a + b \frac{z + z^{-1}}{2}}$ where $b > 0$.

Z-Z-Z

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