PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS) . *I*

BSc DEGREE EXAMINATION MAY 2017
(Fourth Semester)

Branch - MATHEMATICS WITH COMPUTER APPLICATIONS

ANALYTICAL GEOMETRY OF 3P & VECTOR CALCULUS

Time : Three Hours Maximum : 75 Marks ' %

SECTION-A *(20* Marks)
Answer ALL questions ' • ,
ALL questions carry EQUAL marks (10x2 = 20)

1 For what values of c the planes x + 2y + 3z + c = 0 and 2x + cy + 6z + 1 = 0 are parallel.

2 • Find the angle between the planes x- y + z = 1 and 2x - 3y + z = 7.

3 Find the equation of the line through A (1,0, -2) and parallel to the line joining B(4, 6, 7) and C (-2, 8, 1).

4 Write any point of the line other than (1, 2, 3). .

5 Find the centre and radius of the sphere $2x^2 + 2y^2 + 2z^2 + 5x + 7y + 8z - 1 = 0$.

6 Find the equation of the sphere drawn on the segment joining (1, 5, 6) and (-2, 1, 0) as diameter.

7 Find the directional derivative of <)> = $x^2yz + 4xz^2 + xyz$ at (1, 2, 3) in the direction of 2 i + j - k.

8 If f =xz^3 i-2xyzk + xzk, find curl f at (1,2,0).

9 If S is any closed surface enclosing a.volume v , find jj r .ds.

10 State Green's theorem.

SECTION - B (25 Marks!
Answer ALL Questions
ALL Questions Carry EQUAL Marks (5 x 5 = 25)

11 a Find the equation of the plane which bisects perpendicularly the join of (2, 3, 5) and (5,-2, 7).

’ . O R

b A variable plane passes through the fixed point' ($x_{1?}$ yi, *z{)* and meets the co-ordinate axes at A, B, C. Show that the locus of the centroid of the triangle ABC is $\frac{}{x} + \frac{}{y} + \frac{}{z} = 3$.

12 a Show that the lines $\frac{x-5}{A} = \frac{y-7}{A} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = y - 4 = \frac{z-5}{3}$ are co-planar. Find their common point and the equation of the plane in which they lie.

OR

b Find the volume of the tetrahedran whose vertices are (1, 2, 3),,(2, 3, 5) (-2,-1,2) and (3, 0,-3).

Cont

13 a Prove that the two spheres $x^2 + y^2 + z^2 - 2x - 4y — 4z = 0$ and
$x^2 + y^2 + z^2 + lOx + 2z + 10 = 0$ touch each other and find the point of contact.

OR

b Find the equationof the sphere having the circle x^2 •+ $y^2 + z^2 = 9$;
x - 2y + 2z = 5 for a great circle.

14 a If r is the positive vector of the point P(x, y, z) prove (i) div r =3 (ii)
—▶ —> —> —>
Curl r = o (iii) Vr" = nr$^{11/2}$ r where r = | r |.

OR

b If V<J) = $\vec{yz}$ i + $\vec{zx}$ j + $\vec{xy}$ k , find <|>.

15 a If A = (5xy - $6x^2$) i + (2y - 4x) j, evaluate]A.d^ where C is the curve
y = x in the xy plane from the point (1, 1) to (2, 8).

OR

b State Gauss's divergence theorem and the stoke's theorem.

A

SECTION - C (30 Marks!

Answer any THREE Questions

ALL Questions Carry EQUAL Marks (3x10 = 30)

16 Prove that the origin lies in the acute angle between the planes x + 2y + 2z = 9 and 4x — 3y + 12z + 13 = 0. Find the planes bisecting the angles between them and point out which bisects the acute angle.

17 Find the length and the equations of the shortest distance between the lines
x- 10 = $\frac{y-9}{3-2}$ z + 2 x + 1 =------- and------- $\frac{\quad}{2}$ = $\frac{y-12}{4}$ c= z-5.

18 Find the image of the line x + 5 $\frac{y+7}{}$ = z in the plane 2x —y + z + 3 = 0.

19 Prove that curl curl F = 4 grad div F - V^2F and hence deduce that
curl curl curl curl F = V F if F is solenoidal.

• ?

20 Verify Stoke's theorem for F = (y-z) i+ yz j-xzk where S is the surface bounded by the planes x = 0, x = 1 , y = 0 , y = 1 , z = 0 and z = 1 above the xy plane.

z-z-z **END**