14MCU01

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2018
(First Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

CALCULUS

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 Write the conditions for f(x,y) attains a maximum at (a,b).

2 If x^3+y^3+3axy, find $\frac{dy}{dx}$.

3 Write the Cartesian formula for radius of curvature.

4 Define Evolute.

5 If f(x) is an odd function of x, then prove that $\int_{-a}^{a} f(x)dx = 0$

6 Find the value of $\int_0^{\pi/2} \sin^7 x dx$

7 Evaluate $\int_0^1 \int_0^2 (x^2+y^2)dydx$.

8 Define Triple Integrals.

9 Find the value of $\sqrt{(n+1)}$

10 Prove that $\beta(m,n) = \beta(n,m)$

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Find $\frac{du}{dx}$ when $u=x^2+y^2$ where $y = \frac{1-x}{x}$.

OR

b Find the maximum or minimum values of $2(x^2-y^2)-x^4+y^4$.

12 a Find the envelope of the circles drawn on the radius vectors of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ as diameter.

OR

b Prove that the radius of curvature at the point $x=3a\cos\theta-a\cos3\theta$, $y=3a\sin\theta-a\sin3\theta$ is $3a\sin\theta$.

13 a Prove that $\int_0^{\pi/4} \log(1+\tan\theta)d\theta = \frac{\pi}{8}\log 2$

OR

b Evaluate $\int x^3 \cos 2x dx$.

14 a Evaluate $\iint (x^2+y^2)dxdy$ over the region for which x,y are each ≥ 0 and $x+y \leq 1$

OR

b By changing the order of integration, evaluate $\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y} dxdy$.

15 a Evaluate $\int_0^\infty e^{-x^2}\,dx$.

OR

b Prove that $\beta(m,n)=\frac{\sqrt{(m)}\sqrt{(n)}}{\sqrt{(m+n)}}$.

SECTION - C (30 Marks)

Answer any **THREE** Questions
ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 A tent having the form of a cylinder surmounted by a cone is to contain a given volume. If the canvas required is minimum, show that the altitude of the cone is twice that of the cylinder.

17 Show that the evolute of the cycloid x=a(θ-sin θ); y=a(1-cos θ) is another cycloid.

18 If $I_{m,n}=\int x^m(\log x)^n\,dx$ prove that $I_{m,n}=(\log x)^n\frac{x^{m+1}}{m+1}-\frac{n}{m+1}I_{m,n-1}$ and hence evaluate $\int x^4(\log x)^3\,dx$

19 Evaluate $\iiint xyz\,dx\,dy\,dz$ taken through the positive octant of the sphere $x^2+y^2+z^2=a^2$.

20 Evaluate the integral $\iint x^p y^q\,dy\,dx$ over the triangle x>0, y>0, x+y≤1 in terms of Gamma functions.

Z-Z-Z END