PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2018
(Fourth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ANALYTICAL GEOMETRY OF 3D & VECTOR CALCULUS

Time : Three Hours   Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks   (10 x 2 = 20)

1 State the normal form of the equation of a plane.

2 Find the equation of the plane passing through the intersection of the planes $x + y + z = 6$ and $2x + 3y + 4z + 5 = 0$ and the point (1, 1, 1).

3 Find the equation of the line which passes through the point (2, 1, 1) and intersects the lines $2x + y - 4 = 0 = y + 2z$; $x + 3z = 4$, $2x + 5z = 8$.

4 Find the perpendicular distance of the point p(1, 2, 3) from the line $(x - 6)/3 = (y - 7)/2 = (z - 7)/-2$.

5 Find the radius and centre of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z = 2$.

6 Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point (1, 2, 3).

7 Find the directional derivative of $x^2yz + 4xz^2 + xyz$ at (1, 2, 3) in the direction of vector $2i + j - k$.

8 Prove that $A = 3y^4z^2i + 4x^2z^2j - 3x^2y^2k$ is solenoidal.

9 State Green's theorem in the plane.

10 If $A = (5xy - 6x^2)i + (z^2 - 4x)j$, evaluate $\int A\, dx$ where c is the curve $y = x^3$ in the xy - plane from the point (1, 1) to (2, 8).

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Find the equation of the plane through the points P(2, 2, -1), Q(3, 4, 2), R(7, 0, 6).

OR

b Find the area of the triangle whose vertices are the points (1, 2, 3), (-2, 1, -4), (3, 4, -2).

12 a Find the image of the point p(1, 3, 4) in the plane $2x - y + z + 3 = 0$.

OR

b Show that the lines $(x + 3)/2 = (y + 5)/3 = (z - 7)/(-3)$, $(x+1)/4 = (y+1)/3 = (z+1)$ (-1) are coplanar, and then find the equation of the plane containing them.

13 a Show that the two circles $x^2 + y^2 + z^2 - y + 2z = 0$, $x - y + z - 2 = 0$, and $x^2 + y^2 + z^2 + x - 3y + z - 5 = 0$, $2x - y + 4z - 1 = 0$ lie on the same sphere and find its equation.

OR

b Find the equation of the sphere which touches the sphere $x^2 + y^2 + z^2 - x + 3y + 2z - 3 = 0$ at the point (1, 1, -1) and passes through the origin.

Cont ...

14 a Find the angle between the surfaces $Z = x^2 + y^2 - 3$ and $x^2 + y^2 + z^2 = 9$ at (2, -1, 2).

OR

b Prove that div curl F = 0.

15 a Verify Stoke's theorem for $F = (2x - y)i + yz^2j - y^2zk$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and c is the circular boundary on z = 0 plane.

OR

b Prove that the area bounded by a simple closed curve C is given by $\frac{1}{2}\int xdy - ydx$. Hence find the area of an ellipse.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Find the equation of the plane through the points (2, 2, 1) and (9, 3, 6) and perpendicular to the plane 2x + 6y + 6z = 9.

17 Find the magnitude and the equation of the lines of shortest distance between the lines (x – 8)/3 = (y+9) (-16)= (z-10) / 7.
(x-15)/3 = (y-29)/8 = (z-5) / (-5).

18 Find the equations of the sphere through the points (0, 0, 0), (0, 1, -1), (-1, 2, 0), (1, 2, -3).

19 Determine f(r) so that the vector (f(r)) r is both solenoidal and irrotational.

20 Verify the Gauss divergence theorem for $F = xi + y^2j + z^2k$ taken over the cube bounded by planes x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.

Z-Z-Z END