PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2018
(Sixth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

LINEAR ALGEBRA

Time : Three Hours | Maximum : 75 Marks

SECTION-A (20 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 If $P = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$, prove that $P^tP = PP^t$.

2 Define the rank of a matrix A.

3 Define a vector space.

4 Prove that $W^{\perp}$ is a subspace of V, if W is a subset of a vector space V.

5 Define the extension of a field F and the degree of the extension field.

6 State Remainder theorem.

7 Define an algebra.

8 Define a characteristic root and a characteristic vector.

9 Define similar linear transformations.

10 Prove that $T \in A(V)$ is unitary if and only if $TT^* = 1$.

SECTION - B (25 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Show that $U = \frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$ is an unitary matrix.

OR

b Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 4 & 2 \end{bmatrix}$.

12 a If V is a vector space over F, prove the following:
(i) $\alpha o = o$ for $\alpha \in F$ (ii) $ov = o$ for $v \in V$
(iii) $(-\alpha)v = -(\alpha v)$ for $\alpha \in F$, $v \in V$
(iv) if $v \neq 0$, then $\alpha v = 0$ implies that $\alpha = 0$.

OR

b If $u \in V$ and $\alpha \in F$, prove that $||\alpha u|| = |\alpha|\, ||u||$.

13 a If a, b in k are algebraic over F, prove that $a \pm b$, ab, and a/b (if $b \neq 0$) are all algebraic over F.

OR

b If P(x) is a polynomial in F[x] of degree $n > 1$ and is irreducible over F, prove that there is an extension E of F, such that $[E : F] = n$, in which p(x) has a root.

Cont ...

14 a If V is finite – dimensional over F, prove that $T \in A(V)$ is regular if and only if T maps V onto V.

OR

b If $\lambda_1,\ldots,\lambda_k$ in F are distinct characteristic roots of $T \in A(V)$ and $v_1,\ldots,v_k$ are characteristic vectors of T belonging to $\lambda_1,\ldots,\lambda_k$, respectively, prove that $v_1,\ldots,v_k$ are linearly independent over F.

15 a If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, prove that T satisfies a polynomial of degree n over F.

OR

b If $T \in A(V)$ is such that $(vT, v) = 0$ for all $v \in V$, prove that $T = 0$.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Verify Cayley – Hamilton theorem for A and hence find A^{-1}, where

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}.$$

17 If V is a finite – dimensional and if W is a subspace of V, prove that W is finite – dimensional, $\dim W \leq \dim V$, and $\dim V/W = \dim V - \dim W$.

18 If L is a finite – extension of K and if K is a finite extension of F, prove that L is a finite extension of F, and $[L : F] = [L : K][K : F]$.

19 If A is an algebra, with unit element, over F, prove that A is isomorphic to a sub algebra of A(V) for some vector space V over F.

20 If $T \in A(V)$ has all its characteristic roots in F, prove that there is a basis of V in which the matrix of T is triangular.

Z-Z-Z END