

Branch – MATHEMATICS WITH COMPUTER APPLICATIONS

ABSTRACT ALGEBRA

Time : Three Hours

Maximum : 75 Marks

SECTION-A (20 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (10 x 2 = 20)

- 1 Define an Abstract Algebra.
- 2 Define a Subgroup.
- 3 Define an Isomorphism.
- 4 Define a Automorphism.
- 5 Define a Division rings.
- 6 Define a Ideal ring.
- 7 Define a Relatively prime.
- 8 Define a Prime element.
- 9 Define a Primitive.
- 10 Define a unique factorization domain.

SECTION - B (25 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 5 = 25)

- 11 a Show that there is a one-to-one correspondence between any two right cosets of H in G.
OR
b Prove that HK is a subgroup of G if $HK=KH$.
- 12 a State and prove Cayley's theorem.
OR
b Prove that every permutation is the product of its cycles.
- 13 a If P is a prime number then show that J_p the ring of integers mod P , is a field.
OR
b If ϕ is a homomorphism of R into R^1 then prove that
(i) $\phi(0) = 0$ (ii) $\phi(-a) = -\phi(a)$ for every $a \in R$
- 14 a State and prove the Fermat theorem.
OR
b If R is a commutative ring with unit element and M is an ideal of R then prove that M is a maximal ideal of R if R/M is a field.
- 15 a State and prove Eisenstein criterion theorem?
OR
b Prove that if R is a unique factorization domain so is $R[x]$.

SECTION - C (30 Marks)

Answer any THREE Questions

ALL Questions Carry EQUAL Marks (3 x 10 = 30)

- 16 Prove that the subgroup N of G is a normal subgroup of G iff every left coset of N in G is a right coset of N in G .
- 17 State and prove Cauchy's theorem for abelian groups.
- 18 If U is an ideal of the ring R then show that R/U is a ring and is a homomorphic image of R .
- 19 State and prove unique Factorization theorem.
- 20 If $f(x), g(x)$ are two non-zero elements of $F(x)$ then show that $\deg.(f(x)g(x)) = \deg f(x) + \deg g(x)$.