PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2019
(Second Semester)

Branch – **STATISTICS**

PROBABILITY & DISTRIBUTIONS-I

Time: Three Hours　　　　　　　　　　Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks　　　(10 x 1 = 10)

1 The chance that a leap year selected at random will contain 53 Sundays is
(i) 5/7　　(ii) 2/7
(iii) 1/7　　(iv) 3/7

2 For any 2 events A and B, if $B \subset A$ then $P(A \cap \overline{B})$ is equal to
(i) $P(A) - P(B)$　　(ii) $P(A) + P(B)$
(iii) $P(A)\,P(B)$　　(iv) $P(A) / P(B)$

3 The mathematical expression for computing the expected value of a continuous random variable with p.d.f f(x) is
(i) $\int_{-\infty}^{\infty} f(x)dx$　　(ii) $\int_{0}^{\infty} f(x)dx$
(iii) $\int_{-\infty}^{\infty} x\ f(x)dx$　　(iv) $\int_{0}^{1} x\ f(x)dx$

4 If F is the distribution function of the random variable X and if a < b, then $P(a < x \leq b)$ is equal to
(i) $F(b) / F(a)$　　(ii) $F(b)\,F(a)$
(iii) $F(b) + F(a)$　　(iv) $F(b) - F(a)$

5 Mean of Bernoulli distribution with parameter 'P' is
(i) P　　(ii) P^2
(iii) P^3　　(iv) P^4

6 The conditional probability density function of X given Y is defined by
(i) $g(y/x) = \frac{f(x,y)}{h(y)}, h(y) > 0$　　(ii) $g(x/y) = \frac{f(x,y)}{h(y)}, h(y) > 0$
(iii) $g(y) = f(x, y)$　　(iv) $g(x) = f(x, y)$

7 If X is a random variable having probability function f(x), then the function $\sum e^{itx} f(x)$, for 'i' to be an imaginary unit, is known as
(i) Moment generating function　(ii) Probability generating function
(iii) Cumulant generating function　(iv) Characteristic function

8 Let $M_X(t)$ be the moment generating function of a random variable X, then subject to the convergence of the expansion of log $M_X(t)$ in the powers of t, then function : $K_X(t) = \log_e M_X(t)$ is known as
(i) Maclaurin's expansion function　(ii) Probability distribution function
(iii) Moment generating function　(iv) Cumulant generating function

Cont...

9 The expected value of X is equal to the expectation of the conditional expectation of X given Y, symbolically is

(i) E(X) = E[E[X/Y]] (ii) E(X) = E(Y)

(iii) E(X) = E[X/Y] (iv) E(X) = E[E[Y]]

10 If the cumulative distribution function of a continuous random variable X is F(x), then the cumulative distribution function of Y = X + a is.

(i) F(x+a) (ii) F(x-a)

(iii) F(x) (iv) a.F(x)

SECTION - B (25 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a State and prove addition theorem on probability.

OR

b The probabilities of 3 students A. B C solving a problem in statistics are ½, $\frac{1}{3}$, $\frac{1}{4}$. A problem is given to all the 3 students. What is the probability that (i) No one will solve the problem (ii) Only one will solve the problem (iii) Atleast one will solve the problem?

12 a A random variable X has the following probability function :

x :	0	1	2	3	4	5	6	7
P(x) :	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2 + K$

(I) Find K (ii) P(X<6).

OR

b A coin is tossed until a head appears. What is the expectation of the number of tosses?

13 a Given the joint p.d.f of (X, Y) as $f(x, y) = \begin{cases} \frac{K}{(1+x+y)^3}, & x > 0, y > 0 \\ & \text{otherwise} \end{cases}$, Find K.

OR

b Test whether X and Y are independent random variables given that $f(x,y) = \begin{cases} 4xy & 0 < x < 1,\ 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$.

14 a Prove that sum of two independent random variable is equal to the product of their moment generating function.

OR

b State the prove Bernoulli's weak law of large numbers.

15 a If X and Y are independent, prove that E[X / Y] = E[X].

OR

b Let (X, Y) be a two dimensional continuous random variable with f(x,y) = 8xy, 0<y<x<1, find E[Y/X].

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a State and prove Baye's theorem.

OR

b The first of three urns contains 7 white and 10 black balls, the second contains 5 white and 12 black balls and the third contains 17 white balls and no black balls. A person urn at random and draws a ball from it. The ball is white. Find the probabilities that the ball comes from (i) the first urn and (ii) the second urn.

17 a Given the following bivariate probability distribution, obtain (i) marginal distribution of X and Y (ii) Conditional distribution of X when Y = 2.

↓Y / X→	-1	0	1
0	1/15	2/15	1/15
1	3/15	2/15	1/15
2	2/15	1/15	2/15

OR

b Define cumulative distribution function. Prove any two properties of distribution function.

18 a For the joint distribution :

$$f(x, y) = \begin{cases} \frac{9}{4} - x - y, & 0 \le x \le 2,\ 0 \le y \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

Obtain the marginal and conditional distribution of X given Y.

OR

b Explain Bivariate distribution.

19 a State and prove Tchebychev's inequality.

OR

b Explain weak law of large numbers.

20 a Given the joint density function of X and Y is $f(x,y) = \begin{cases} \frac{1}{2} x e^{-y}, & 0 < x < 2,\ y > 0 \\ 0 & \text{otherwise} \end{cases}$, find the distribution of X + Y.

OR

b Explain transformation of variables with an example.

Z-Z-Z END