14MCU25

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2019
(Sixth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

COMPLEX ANALYSIS

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 Define continuity of f(z) at z_0.
2 Show that $f(z) = \bar{z}$ is nowhere differentiable.
3 Define critical point of f(z).
4 If f(z) is analytic function, then prove that $\frac{\partial(u,v)}{\partial(x,y)} = |f'(z)|^2$.
5 Evaluate $\int_c \frac{z}{z-2} dz$ where C is the circle $|z| = 1$.
6 State Cauchy's theorem.
7 Find the Zeros of $f(z) = \frac{z^3-1}{z^3+1}$.
8 State Schwarz lemma.
9 State Jordan's lemma.
10 Find the residue of cot z at z = 0.

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Show that $u = \frac{1}{2}\log(x^2+y^2)$ is harmonic and find its conjugate.

OR

b Show that an analytic function in a region with constant modules is constant.

12 a i) Define the inverse of a point with respect to a circle.
ii) Write any three elementary transformation.

OR

b Find the in inverse point of the infinite strips $\frac{1}{4} < y < \frac{1}{2}$ under the transformation $w = \frac{1}{2}$.

13 a Evaluate $\int \frac{zdz}{z^2-1}$ where C is positively oriental circle $|z| = 2$.

OR

b State and prove Morera's theorem.

14 a State and prove Liouville's theorem.

OR

b State and prove maximum modulus principle.

Cont ...

15 a State and prove Cauchy's Residue theorem.

OR

b Find the residue of $\frac{1}{(z^2+1)^3}$ at $z = i$.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 State and prove Cauchy – Riemann equation.

17 Let f(z) be an analytic function of z in a region D of the Z-plane and let $f^1(z) \neq 0$ inside D. Prove that the mapping $w = f(z)$ is conformal at the points of D.

18 State and prove Poisson's integral formula of a circle.

19 State and prove Taylor's theorem.

20 Evaluate $\int_0^\infty \frac{x^2 dx}{x^6+1}$.

Z-Z-Z END