**18MCU01**

**PSG COLLEGE OF ARTS & SCIENCE**
(AUTONOMOUS)
**BSc DEGREE EXAMINATION MAY 2019**
(First Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

**CALCULUS**

Time: Three Hours                                        Maximum: 75 Marks

**SECTION-A (10 Marks)**
Answer **ALL** questions
**ALL** questions carry **EQUAL** marks                (10 x 1 = 10)

1 If $\lim_{t\to a} \vec{r}(t) = \vec{r}(a)$ then the vector function $\bar{r}$ is ____.
(i) continuous  (ii) continuous at a
(iii) differentiable  (iv) differentiable at a

2 Unit tangent vector T(t) is equal to
(i) $\frac{r'(t)}{|r'(t)|}$  (ii) $\frac{r(t)}{|r'(t)|}$  (iii) $\frac{r'(t)}{|r(t)|}$  (iv) $\frac{r(t)}{|r(t)|}$

3 The Laplace equation of the function u is _______.
(i) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=u$  (ii) $\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}=u$
(iii) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$  (iv) $\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}=0$

4 If $f(x,y)=4-x^2-2y^2$, then fx(1,1) is equal to ______.
(i) -2  (ii) -4  (iii) 2  (iv) 4

5 A function of two variables has a local maximum at (a,b) if _____ when (x,y) is near (a,b).
(i) f(x,y)<f(a,b)  (ii) f(x,y)>f(a,b)  (iii) f(x,y)≥f(a,b)  (iv) f(x,y)≤f(a,b)

6 If $f(x,y)=\sin x+e^{xy}$ then $\nabla f(0,1)$=_____.
(i) 2  (ii) 0  (iii) (2,0)  (iv) (0,2)

7 The moment of the entire lamina about the x axis is
(i) $M_x = \iint_D x\rho(x,y)dA$  (ii) $M_x = \iint_D y\rho(x,y)dA$
(iii) $M_x = \iint_D x^2\rho(x,y)dA$  (iv) $M_x = \iint_D y^2\rho(x,y)dA$

8 The moment of inertia about the origin is
(i) 0  (ii) $I_x$  (iii) $I_y$  (iv) $I_x+I_y$

9 The triple integral is used to find the _________.
(i) length  (ii) area  (iii) volume  (iv) density

10 For the cylindrical coordinates $(2, 2\pi/3, 1)$ the corresponding rectangular coordinates are
(i) $(-1,\sqrt{3},1)$  (ii) $(1,\sqrt{3},1)$  (iii) $(-1,\sqrt{3},-1)$  (iv) $(-1,-\sqrt{3},-1)$

**SECTION - B (25 Marks)**
Answer **ALL** questions
**ALL** questions carry **EQUAL** Marks    (5 x 5 = 25)

11 a If $\bar{r}(t) = 2\cos t\, i + \sin t\, j + 2\, t\, k$, find $\int_0^{\pi/2} \bar{r}(t)dt$.

OR

b Find the curvature of the twisted cube $\bar{r}(t) < t, t^2, t^3 >$ at a general point and at (0, 0, 0).

13 a If $u = x^4 y + y^2 z^3$, where $x = r s e^t$, $y = r s^2 e^{-t}$ and $z = r^2$,s.sint, find $\frac{\partial u}{\partial s}$ when r = 2, s = 1, t = 0.

OR

b Define local minimum, local maximum and saddle point.

14 a Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ and above the region D in the xy – plane bounded by the line y = 2x and the parabola $y = x^2$.

OR

b Evaluate $\iint_R (3x + 4y^2)dA$, where R is the region in the upper half-plane bounded by the circles $x^2+y^2 = 1$ and $x^2 + y^2 = 4$.

15 a Evaluate $\iiint_E z\,dv$, where E is the solid tetrahedron bounded by the four planes x = 0, y = 0, z = 0 and x + y + z = 1.

OR

b Given the point with cylindrical co-ordinate $(2, 2\frac{\pi}{3}, 1)$, find its rectangular co-ordinates.

**SECTION -C (40 Marks)**

Answer **ALL** questions

**ALL** questions carry **EQUAL** Marks (5 x 8 = 40)

16 a Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane y + z = 2.

OR

b i) Find the length of th arc of the circular helix with vector equation $\bar{r}(t) = \cos t\, i + \sin t\, j + t\, k$, from the point (1, 0, 0) to the point (1, 0, 2π).

ii) Show that the curvature of a circle of radius λ is $1/\lambda$.

17 a Verify that the function Z = $I_n$ $(e^x + e^y)$ is a solution of $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = .1$ and $\frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 = 0$.

OR

b Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at (1, 1, 3).

18 a Find the directional derivative $D_u f(x, y)$ if $f(x, y) = x^3 - 3xy + 4y^2$ and $\bar{u}$ is the unit vector given by angle $\theta = \frac{\pi}{6}$. What is $D_u f(1,2)$?

OR

b Find the local maximum and minimum values and saddle points of $f(x, y) = x^4 + y^4 - 4xy + 1$.

19 a Find the volume of tetrahedron bounded by the planes x + 2y + z = 2, x = 2y, x = 0 and z = 0.

OR

b Find the mass and center of mass of a triangular lamina with vertices (0, 0), (1, 0) and (0, 2), if the density function is $\rho(x, y) = 1 + 3x + y$.

20 a Evaluate the triple integral $\iiint_B xyz^2 dv$, where B is the rectangular box given by $B = \left\{ (x, y, z) / 0 \le x \le 1, -1 \le y \le 2, 0 \le z \le 3 \right\}$.

OR