PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION MAY 2019
(Second Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ANALYTICAL GEOMETRY OF 3D & VECTOR CALCULUS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 Indicate the intercept form of the equation of a plane.
(i) $\frac{x}{b}+\frac{y}{c}+\frac{z}{a}=1$ (ii) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ (iii) $\frac{x}{b}+\frac{y}{c}+\frac{z}{a}=0$ (iv) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$

2 Find the equation to the plane through (3,4,5) parallel to the plane 2x+3y-z=0.
(i) 2x+3y-z-13=0 (ii) 2x-3y-z+13=0
(iii) 2x+3y+z+13=0 (iv) 2x-3y+z-13=0

3 The condition for the line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ to be lies in the plane ax+by+cz+d=0 is
(i) al+bm+cn=0 and $ax_1+by_1+cz_1+d\neq0$
(ii) al+bm+cn≠0 and $ax_1+by_1+cz_1+d=0$
(iii) al+bm+cn≠0 and $ax_1+by_1+cz_1+d\neq0$
(iv) al+bm+cn=0 and $ax_1+by_1+cz_1+d=0$

4 When the two lines $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$ are coplanar if the shortest distance between them is
(i) zero (ii) unity (iii) either zero or unity (iv) neither zero nor unity

5 What is the equation of the sphere, when the centre of the sphere is at origin and its radius is a
(i) $x^2+y^2+z^2=a$ (ii) $x^2+y^2+z^2=0$
(iii) $x^2+y^2+z^2=a^2$ (iv) $(x-a)^2+(y-b)^2+(z-c)^2=a^2$

6 Find the equation of the tangent plane to the sphere $x^2+y^2+z^2=r^2$ at the point (x_1,y_1,z_1) is
(i) $xy+x_1y_1+zz_1=r^2$ (ii) $xx_1+yy_1+zz_1=r^2$
(iii) $xx_1+yy_1+zz_1=r$ (iv) $xx_1+yy_1+zz_1=0$

7 Suppose $\vec{F}$ is a vector field that is continuous on an open connected region D. If $\int_C \vec{F}.d\vec{r}$ is independent of path in D, then $\vec{F}$ is a
(i) vector field on D (ii) conservative field on D
(iii) conservative vector field on D (iv) None of the above

8 If $\vec{F}=P\hat{i}+Q\hat{j}+R\hat{k}$ is a vector field on $\mathbb{R}^3$ and P, Q and R have continuous second-order partial derivatives, then div curl $\vec{F}=$
(i) 1 (ii) Curl $\vec{F}$ (iii) div $\vec{F}$ (iv) 0

9 Find the parametric equations for the cylinder. $x^2+y^2=4$, $0\leq z\leq1$
(i) x=2 cosθ, y=2 sinθ, z=z (ii) x=2 sinθ, y=2 cosθ, z=z
(iii) x=sinθ y=2 cosθ z=3z (iv) x=sinθ y=cosθ z=2z

10 Find the tangent vector $\vec{r}_u$ to the surface with parametric equations $x=u^2$, $y=v^2$, z=u+2v.
(i) $2u\hat{i}+\hat{k}$ (ii) $2u\hat{i}+\hat{j}+\hat{k}$ (iii) $2v\hat{j}+\hat{k}$ (iv) $\hat{i}+\hat{j}+\hat{k}$

SECTION - B (25 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Develop the equation of the plane passing through the points (3,1,2), (3,4,4) and perpendicular to the plane $5x+y+4z=0$.

OR

b Find the equation of the plane through the point (1,-2,3) and the intersection of the planes $2x-y+4z=7$ and $x+2y-3z+8=0$.

12 a Calculate the image of the point (1,-2,3) in the plane $2x-3y+2z+3=0$.

OR

b Find the condition for the lines $ax+by+cz+d=0=a_1x+b_1y+c_1z+d_1$; $a_2x+b_2y+c_2z+d_2=0=a_3x=b_3y+c_3z+d_3$ to be coplanar.

13 a Find the equation of the sphere which has its centre at the point (6,-1,2) and touches the plane $2x-y+2z-2=0$.

OR

b Find the equation of a sphere which touches the sphere $x^2+y^2+z^2-6x+2z+1=0$ at the point (2,-2,1) and passes through the origin.

14 a Evaluate $\int_C 2x\,ds$, where C consists of the arc C_1 of the parabola $y=x^2$ from (0,0) to (1,1).

OR

b Find the work done by the force field $\vec{F}(x,y)=x^2\hat{i}-xy\hat{j}$ in moving a particle along the quarter-circle $\vec{r}(t)=\cos t\hat{i}+\sin t\hat{j},\ 0\le t\le \pi/2$.

15 a Find the area of the part of the paraboloid $z=x^2+y^2$ that lies under the plane $z=9$.

OR

b Find the flux of the vector field $\vec{F}(x,y,z)=z\hat{i}+y\hat{j}+x\hat{k}$ over the unit sphere $x^2+y^2+z^2=1$.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a Find the equation of the plane passing through the points (2,-5,-3), (-2,-3,5) and (5,3,-3).

OR

b Show that the origin lies in the acute angle between the planes $x+2y+2z=0$, $4x-3y+12z+13=0$. Find the planes bisecting the angles between them and point out which bisects the obtuse angle.

17 a Prove that the lines $\frac{x+1}{-3}=\frac{y+10}{8}=\frac{z-1}{2}$; $\frac{x+3}{-4}=\frac{y+1}{7}=\frac{z-4}{1}$ are coplanar. Find also their point of intersection and the plan through them.

OR

b Find the locus of the mid-points of lines whose extremities are on two given lines and which are parallel to a given plane.

18 a Find the equation to the sphere through the four the circle (2,3,1), (5,-1,2), (4,3,-1) and (2,5,3).

OR

b Find the equation of the sphere which passes through the circle $x^2+y^2+z^2-2x-4y=0$, $x+2y+3z=8$ and touches the plane $4x+3y=25$.

19 a Evaluate $\int_C y\,dx+z\,dy+x\,dz$, where C consists of the lines. Segment C_1 from (2,0,0) to (3,4,5) followed by the vertical line segment C_2 from (3,4,5) to (3,4,0).

OR

b If $\vec{F}(x,y)=\frac{(-y\hat{i}+x\hat{j})}{x^2+y^2}$, show that $\int_C \vec{F}.d\vec{r}=2\pi$ for every positively oriented simple closed path that encloses the origin.

20 a Find the surface of a sphere of radius a.

OR

b Verify the divergence theorem for the vector field $\vec{F}$ on the region E, where $\vec{F}(x,y,z)=3x\bar{i}+xy\bar{j}+2xz\bar{k}$ and E is the cube bounded by the planes ...