**PSG COLLEGE OF ARTS & SCIENCE**
(AUTONOMOUS)

**BSc DEGREE EXAMINATION MAY 2019**
(First Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

# ORDINARY DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORMS

Time: Three Hours | Maximum: 75 Marks

## SECTION-A (10 Marks)

Answer **ALL** questions
**ALL** questions carry **EQUAL** marks (10 x 1 = 10)

1 A homogenous first order differential equation $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ can be transformed into a separable equation of the form __________.

(i) $\frac{dv}{dx} = xf(v) - v$ (ii) $x\frac{dv}{dx} = f(v) - v$
(iii) $\frac{dv}{dx} = f(v) - v$ (iv) $x\frac{dv}{dx} = f(v)$

2 The differential equation $\frac{dy}{dx} = \frac{y^2}{1-xy}$ is __________.

(i) an exact differential equation (ii) not an exact differential equation
(iii) a partial differential equation (iv) homogenous linear differential equation

3 The Wronskian w(x) of cos x and sin x is __________.

(i) 1 (ii) 0
(iii) cos x + sin x (iv) cos x – sin x

4 The general solution of $2y'' - 7y' + 3y = 0$ is __________.

(i) $C_1e^{x/2} + C_2xe^{3x}$ (ii) $C_1e^{2x} + C_2e^{3x}$
(iii) $C_2e^{x/2} + C_2e^{3x}$ (iv) $2C_1e^x + 3C_2e^x$

5 The polar form of a complex number is __________.

(i) $x + iy = re^{\theta}$ (ii) $x + iy = re^{i\theta}$
(iii) $x = re^{i\theta}$ (iv) $x + iy = e^{i\theta}$

6 The reactance of the circuit is __________.

(i) S = WL - WC (ii) $Z = \sqrt{R^2 + \left(WL - \frac{1}{WC}\right)^2}$
(iii) $Z = R^2 + \left(WL - \frac{1}{WC}\right)^2$ (iv) $S = WL - \frac{1}{WC}$

7 $\Gamma\left(\frac{5}{2}\right) =$ __________.

(i) $\sqrt{\pi}$ (ii) $\frac{1}{4}\sqrt{\pi}$
(iii) $\frac{3}{4}\sqrt{\pi}$ (iv) $\frac{1}{2}\sqrt{\pi}$

8 $L(\sin k t) =$ ____________.

(i) $\frac{k}{s^2+k^2}$ (ii) $\frac{s}{s^2+k^2}$ (iii) $\frac{k}{s^2-k^2}$ (iv) $\frac{s}{s^2-k^2}$

9 The convolution $(f * g)$ of a piecewise continuous functions f and g for $t \geq 0$ is ____________.

(i) $\int_0^t f(\tau)g(t-\tau)d\tau$ (ii) $\int_0^\tau f(\tau)g(t-\tau)d\tau$

(iii) $\int_0^t f(t-\tau)g(t-\tau)d\tau$ (iv) $\int_0^t f(t-\tau)g(t)d\tau$

10 $L^{-1}(e^{ax}) =$ __________.

(i) $\frac{1}{s+a}$ (ii) $\frac{1}{a-s}$ (iii) $\frac{1}{s-a}$ (iv) $\frac{a}{s-a}$

## SECTION - B (25 Marks)

Answer **ALL** questions

**ALL** questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Solve the differential equation $\frac{dy}{dx} = (x+y+3)^2$.

OR

b Solve the equation $xy'' + 2y' = 6x$ in which the dependent variable y is missing.

12 a Let $y_1$ and $y_2$ be two solutions of the homogenous linear equation $y'' + p(x)y' + q(x)y = 0$ on the interval I, If $C_1$ and $C_2$ are constants, then prove that the linear combination of $y_1$ & $y_2$ is also a solution.

OR

b Show that the functions $f(x) = x$, $g(x) = xe^x$, $h(x) = x^2e^x$ are linearly independent.

13 a Solve the initial value problem $y^{(3)} + 3y^{(2)} - 10y^{(1)} = 0$, $y(0) = 7$, $y'(0) = 0, y''(0) = 70$.

OR

b Find a particular solution of $3y'' + y'y - 2y = 2\cos x$.

14 a Find the Laplace transform of $t^2 + \cos 2t \cos t + \sin^2 2t$.

OR

b Find $L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right]$.

15 a A mass that weighs 32lb is attached to the free end of a long light spring that is stretched 1 ft by a force of 4 lb. The mass is initially at rest in its equilibrium position. Beginning at time t = 0, an external force f(t) = cos 2 t is applied to the mass, but at time t = 2 π this force is turned off and the mass is allowed to continue its motion unimpeded. Find the resulting position function x(t) of the mass.

OR

b Let f(t) be periodic with period p and piecewise continuous for $t \geq 0$. Then the transform F(S) = L{f(t)} exists for s > 0 and is given by

$F(s) = \frac{1}{\ldots}\int_0^p e^{-st} f(t)dt$

**SECTION -C (40 Marks)**

Answer **ALL** questions

**ALL** questions carry **EQUAL** Marks (5 x 8 = 40)

16 a i) Solve the initial value problem $x\frac{dy}{dx} = y + \sqrt{x^2 - y^2}, y(x_0) = 0, x_0 > 0$.

ii) Solve $2xe^{2y}\frac{dy}{dx} = 3x^4 + e^{2y}$.

OR

b Derive the equation of the plane's trajectory.

17 a Verify that the functions $y_1(x) = e^x$ and $y_2(x) = xe^x$ are solutions of the differential equations $y'' - 2y' + y = 0$ and find a solution satisfying the initial conditions y(0)=3, $y'(0) = 1$.

OR

b Show that the three solutions $y_1(x) = e^x, y_2(x) = e^{-x}, y_3(x) = e^{-2x}$ of the third order equation $y''' + 2y'' - y' - 2y = 0$ are linearly independent. Also find a particular solution satisfying the initial conditions y(0)=1, $y'(0) = 2, y''(0) = 0$.

18 a A body with mass m = ½ kg is attached to the end of a spring that is stretched 2m by a force of 100 newtons. It is set in notion with initial position x0 = 1m and initial velocity $v_0$ = -5 m/s. Find the position function of the body as well as the amplitude, frequency, period of oscillation and time lag of its motion.

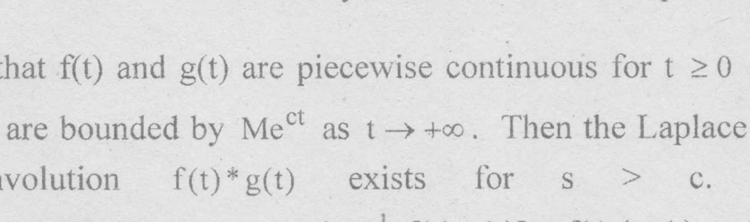
OR

b Find a particular solution of $y'' + 4y = 3x^3$.

19 a Solve the initial value problem $x'' - x' - 6x = 0$, x(0)=2, $x'(0) = -1$.

OR

b Solve the system $2x'' = -6x + 2y, y'' = 2x - 2y + 40 \sin 3t$ subject to the initial condition $x(0) = x'(0) = y(0) = y'(0) = 0$.




Thus the force f(t) = 40 sin 3t is suddenly applied to the second mass of the above figure at the time t = 0, when the system is at rest in its equilibrium position.

20 a Suppose that f(t) and g(t) are piecewise continuous for $t \geq 0$ and that $|f(t)|$ and $|g(t)|$ are bounded by $Me^{ct}$ as $t \to +\infty$. Then the Laplace transform of the convolution $f(t) * g(t)$ exists for s > c. Moreover $L[f(t) * g(t)] = L[f(t)]. L[g(t)]$ and $L^{-1}[f(s).g(s)] = f(t) * g(t)$.

OR

b Suppose that f(t) is piecewise continuous for $t \geq 0$, that f(t) satisfies the condition $\lim_{t \to 0^+} \frac{f(t)}{t}$ and that $|f(t)| \leq Me^{Ct}$ as $t \to +\infty$, then

$$L\left\{\frac{f(t)}{t}\right\} = \int^{\infty} f(\sigma)d\sigma \text{ for } s > c. \text{ Equivalently } f(t) = L^{-1}[F(s)] = tL^{-1}\left\{\int^{\infty} f(\sigma)d\sigma\right\}.$$