14MCU12

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2019
(Fourth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ANALYTICAL GEOMETRY OF 3D & VECTOR CALCULUS

Time : Three Hours   Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks   (10 x 2 = 20)

1 Find the direction cosines of the normal to the plane $2x - 3y + 6z = 7$.

2 Obtain the equation of the plane through the point (x_1, y_1, z_1) and parallel to the plane $ax + by + cz + d = 0$.

3 Find the co-ordinates of the point of intersection of the line $\frac{x+1}{1} = \frac{y+3}{3} = \frac{z-2}{-2}$ with the plane $3x + 4y + 5z = 5$.

4 Find the equation of plane containing the coplanar lines $\frac{x+3}{2} = \frac{y+5}{3} = \frac{z-7}{-3}$, $\frac{x+1}{4} = \frac{y+1}{5} = \frac{z+1}{-1}$.

5 Find the radius and centre of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z = 2$.

6 Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point (1, 2, 3).

7 Find the unit tangenet vector for the curve $r = a\cos t\,\bar{i} + a\sin t\,\bar{j} + ct\,\bar{k}$.

8 Prove that curl (grad ϕ) = 0.

9 Evaluate $\int_C A.dr$ where $A = (5xy - 6x^2)\bar{i} + (2y - 4x)\bar{j}$ & C is the curve $y = x^3$ in the xy plane from the point (1, 1) to (2, 8).

10 If $\nabla\phi = 5r^3\bar{r}$, find ϕ.

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Find the equation of the plane through the points (2, 2, 1) and (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$.

OR

b Find the locus of a point, the sum of the square of whose distances from the planes $x + y + z = 0$, $x - z = 0$, $x - 2y + z = 0$ is 9.

12 a Find the equation of the line which passes through the point (2, -1, 1) and intersects the lines $2x + y - 4 = 0 = y + 2z$; $x + 3z = 4$, $2x + 5z = 8$.

OR

b Prove that the planes $x = cy + bz$, $y = az + cx$, $z = bx + ay$ pass through one line $a^2 + b^2 + c^2 + 2abc = 1$. Show that the equations of this line are

$$\frac{x}{\sqrt{1-a^2}} = \frac{y}{\sqrt{1-b^2}} = \frac{z}{\sqrt{1-c^2}}.$$

13 a A sphere of constant radius 2k passes through the origin and meets the axes in A, B, C. Find the locus of the centroid of the tetrahedron OABC.

OR

b Find the equations of the two tangent planes to the sphere $x^2 + y^2 + z^2 = 9$, which passes through the line $x + y = 6$, $x - 2z = 3$.

14 a If $\bar{r} = \bar{a}$ cosh nt + $\bar{b}$ sinh nt where $\bar{a}$, $\bar{b}$, n are constants. Show that $\bar{r} \times \frac{d\bar{r}}{dt} = n(\bar{a} \times \bar{b})$.

OR

b Prove that curl $(\bar{u} \times \bar{v}) = (v.\nabla)\bar{u} - (\bar{u}.\nabla)\bar{v} + \bar{u}$ div v – v div u.

15 a Find the common area between $y^2 = 4x$ and $x^2 = 4y$ by using green's theorem.

OR

b Evaluate $\iint_S (x^3 dydz + x^2 ydzdx + x^2 zdxdy)$. where S is the surface of the cube x = 0, y = 0, z = 0, x = 1, y = 1, z = 1.

SECTION - C (30 Marks)

Answer any **THREE** Questions
ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Show that the origin lies in the acute angle between the planes x + 2y + 2z = 9 and 4x – 3y + 12z + 13 = 0. Find the planes bisecting the angles between them and point out which bisects the acute angle.

17 Find the magnitude and the equations of the line of shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$, $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

18 Find the equation to the sphere through the points (0, 0, 0), (0, 1, -1), (-1, 2, 0), (1, 2, 3).

19 Prove that curl curl F = grand div $\bar{F} - \nabla^2 \bar{F}$ and hence deduce that curl curl curl F = $\nabla^4 \bar{F}$ if $\bar{F}$ is solenoid.

20 Verify stoke's theorem for $\bar{F} = (y = z)\bar{i} + yz\bar{j} - xz\bar{k}$ where S is the surface bounded by the planes x = 0, x = 1, y = 0, y = 1, z = 0 and z = 1 above the xoy plane.

Z-Z-Z END