DIFFERENTIAL EQUATIONS & LAPLACE TRANSFORM

Time: Three Hours   Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions

ALL questions carry **EQUAL** marks   (10 x 1 = 10)

1 The order of a differential equation is the order of the _____ derivative that appears in it.
   (i) Higher   (ii) Lower   (iii) Middle   (iv) None of these

2 The first order differential equation of the form $\frac{dy}{dx} + p(x)\,y = Q(x)\,y^n$ is called
   (i) Exact equation   (ii) Laplace equation
   (iii) Bernoulli's equation   (iv) None of these

3 W (cos x, sinx) is
   (i) 0   (ii) 1   (iii) -1   (iv) None of these

4 The characteristic equation of $y^{(3)} + 3y'' - \log' = 0$ is
   (i) $r^3 + 3r^2 - 10r = 0$ (ii) $r^3 + 3r^2 - 10 = 0$
   (iii) $r^3 + 3r - 10r = 0$ (iv) None of these

5 Complementary function of $y'' - 3y' + 2y = e^{2x}$ is
   (i) $C_1 e^x + c_2 e^{2x}$ (ii) $C_1 + C_2 x^{ex}$ (iii) $C_1 + C_2 x e^{2x}$ (iv) None of these

6 The general _____ n^{th} order linear equation with constant coefficients has the form $a_n y^{(n)} + a_n + y^{(n-1)} + \ldots + a_1 y' + a_0 y = f(x)$.
   (i) Homogeneous   (ii) Exact   (iii) Non homogeneous (iv) None of these

7 L(sinkt)
   (i) $\frac{s}{s^2 + k^2}$ (ii) $\frac{k}{s^2 + k^2}$ (iii) $\frac{s}{s^2 - k^2}$ (iv) $\frac{k}{s^2 - k^2}$

8 $L^{-1}\left(\frac{1}{(s-a)^2}\right)$ is
   (i) $t\,e^{-at}$ (ii) $t\,e^{at}$ (iii) $t^2\,e^{at}$ (iv) None of these

9 If L{f(t)} exists for s > c then L (u(t –a) f(t – a)) is
   (i) $e^{as} f(s)$ (ii) $e^{-as} f(s)$ (iii) $e^{as} f'(s)$ (iv) None of these

10 L{t^n f(t)} is
   (i) $F^n(s)$ (ii) $(-1)^n F^{(n)}(s)$ (iii) $\frac{1}{t} F^n(s)$ (iv) None of these

SECTION - B (25 Marks)
Answer **ALL** questions

ALL questions carry **EQUAL** Marks   (5 x 5 = 25)

11 a Solve the initial value problem $\frac{dy}{dx} = 2x + 3$, y(1) = 2.

OR

b Suppose that at time t = 0, 10 thousand people in a city with population M = 100 thousand people have heard a certain rumor. After 1 week the number p(t) of those who have heard it has increased to p(1) = 20 thousand. Assuming that p(t) satisfies a logistic equation, when will 80% of the city's population have heard the rumor?

12 a Find the general solution of $2y'' - 7y' - 3y = 0$.

OR

b $y_p = 3x$ is a particular solution of $y'' + 4y = 12x$ and $y_c(x) = c_1 \cos 2x + c_2 \sin 2x$ is its complementary function. Find the solution of $y'' + 4y = 12x$ that satisfies $y(0) = 5$, $y'(0) = 7$.

13 a Find a particular solution of $y'' - 4y = 2e^{3x}$.

OR

b Determine the appropriate form for a particular solution of
$y'' + 6y' + 13y = e^{-3x} \cos 2x$.

14 a Find $L\{3e^{2t} + 2\sin^2 3t\}$.

OR

b Show that $L\{t\, e^{at}\} = \dfrac{1}{(s-a)^2}$

15 a Find $L^{-1}\left\{\tan^{-1}\left(\dfrac{1}{s}\right)\right\}$.

OR

b Find $L\{f(t)\}$ if $f(t) = \begin{cases} \cos 2t & \text{if } 0 \leq t < 2\pi \\ 0 & \text{if } t \geq 2\pi \end{cases}$.

SECTION -C (40 Marks)
Answer ALL questions
ALL questions carry EQUAL Marks   (5 x 8 = 40)

16 a Solve $2xy\dfrac{dy}{dx} = 4x^2 + 3y^2$.

OR

b Consider an animal population $p(t)$ that is modeled by $\dfrac{dp}{dt} = 0.0004p\,(p-150)$.
Find $p(t)$ if (i) $p(0) = 200$ (ii) $p(0) = 100$.

17 a Verify that the functions $y_1(x) = e^x$ and $y_2(x) = xe^x$ are solutions of $y'' - 2y' + y = 0$ and then find a solution satisfying the initial conditions $y(0) = 3$, $y'(0) = 1$.

OR

b Find the particular solution of $y'' - 4y' + 5y = 0$ for which $y(0) = 1$ and $y'(0) = 5$.

18 a Solve the initial value problem
$y'' - 3y' + 2y = 3e^{-x} - 10\cos 3x$;
$y(0) = 1$ $y'(0) = 2$.

OR

b Find a particular solution of $y^{(3)} + y'' = 3e^x + 4x^2$.

19 a Solve the initial value problem
$x'' + 4x = \sin 3t$; $x(0) = x'(0) = 0$.

OR

b Solve the initial value problem $y'' + 4y' + 4y = t^2$; $y(0) = y'(0) = 0$.

20 a Find $L^{-1}\left(\dfrac{2s}{(s^2-1)^2}\right)$.

OR

b Consider a mass on a spring with $m = k = 1$ and $x(0) = x'(0) = 0$. At each of the instants $t = 0, \pi, 2\pi, 3\pi, \dots n\pi \dots$ The mass is struck a hammer below with a unit impulse. Determine the resulting motion.

Z-Z-Z   END