14MAU20

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2019
(Sixth Semester)

Branch – **MATHEMATICS**

COMPLEX ANALYSIS

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 When you say a curve is simple?
2 Define a continuous function in the bounded closed domain D..
3 Define Conformal Mapping.
4 Give an example for a transformation which is isogonal but not conformal.
5 When an arc is said to be differentiable?
6 State Gauss's mean value theorem.
7 Define an entire function.
8 When we say the singularity is isolated essential?
9 What is the formula for Residue of $\frac{\phi(z)}{(z-a)^m}$ at z=a?
10 Write Jordan's inequality.

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Prove that read and imaginary parts of an analytic function satisfy Laplace's equation.

OR

b If $u=(x-1)^3-3xy^2+3y^2$, determine v so that u+iv is a regular function of x+iy.

12 a Prove that for the transformation w=z+(1-i) and determine the region D^1 of the w-plane corresponding to the rectangular region D in the z-plane bounded by x=0, y=0, x=1 and y=2.

OR

b Prove that the superficial magnification of the conformal transformation w=f(z) is $|f'(z)|^2$.

13 a Evaluate $\int_L \frac{dz}{z-a}$ where L represents a circle $|z-a|=r$.

OR

b State and prove Cauchy's fundamental theorem.

14 a State and prove Liouville's theorem.

OR

b Expand $f(z)=\frac{1}{(z+1)(z+3)}$ in a Laurent's series valid for the regions

(i) $|z|<1$ (ii) $|z|>3$

Cont...

15 a Find the residue of $\frac{1}{(z^2+1)^3}$ at z=i.

OR

b State and prove Jordan's Lemma.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Derive the polar form of Cauchy-Riemann equations.

17 Derive the necessary conditions for w=f(z) to represent a conformal mapping.

18 State and prove Poisson's integral formula for a circle.

19 For the function $f(z)=\frac{2z^3+1}{z^2+z}$ find

(i) a taylor series valid in the neighbourhood of the point z=i.

(ii) a Laurent's series valid within the annulus of which centre is the region.

20 Show that $\int_0^{2\pi}\frac{d\theta}{a+b\cos\theta}=\int_0^{2\pi}\frac{d\theta}{a+b\sin\theta}=\frac{2\pi}{\sqrt{(a^2-b^2)}}$, a>b>0.

Z-Z-Z END