PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2019
(Second Semester)

Branch – **MATHEMATICS**

CALCULUS-II

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 If $\lim_{n\to\infty} a_n$ exists then the sequence is
(i) divergent (ii) converges (iii) convergent (iv) diverges

2 A geometric series $\sum ar^{n-1}$ converges then mention the limit of r.
(i) $|r|<1$ (ii) $|r|>1$ (iii) $|r|\geq 1$ (iv) $|r|=1$

3 Identify the series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}$
(i) conditionally convergent (ii) absolutely convergent
(iii) divergent (iv) absolutely divergent

4 What is the interval of convergence of the series $\sum_{n=0}^{\infty} x^n$?
(i) [-1, 1) (ii) (-1, 1) (iii) [-1, 1] (iv) (-1, 1]

5 Find the gradient vector field of $f(x, y) = x^2y - y^3$.
(i) $\nabla f(x,y) = 2xy\bar{i} + (x^2 - 3y^2)\bar{j}$ (ii) $\nabla f(x,y) = 2xy\bar{i} - (x^2 - 3y^2)\bar{j}$
(iii) $\nabla f(x,y) = 2xy\bar{j} + (x^2 - 3y^2)\bar{i}$ (iv) $\nabla f(x,y) = 2xy\bar{j} - (x^2 - 3y^2)\bar{i}$

6 Identify a curve that doesn't intersect itself anywhere between its end points.
(i) Transcendental curve (ii) Spiral curve
(iii) Simple (iv) Compound curve

7 Indicate the parametric representations of surfaces are
(i) not to be defined (ii) unique (iii) not unique (iv) none

8 For a closed curve C, mention the Stoke's theorem.
(i) $\int_c F.dr = 2\pi$ (ii) $\int_c F.T\ dr = 0$ (iii) $\int_c F.dr = \infty$ (iv) $\int_c F.dr = 0$

9 What is the expansion of tan (A+B+C+....)?
(i) $\frac{S_1 - S_3 - S_5.....}{1 + S_2 - S_4.....}$ (ii) $\frac{S_1 - S_3 + S_5.....}{1 + S_2 - S_4.....}$
(iii) $\frac{S_1 - S_3 + S_5.....}{1 - S_2 + S_4.....}$ (iv) $\frac{S_1 - S_3 + S_5.....}{1 + S_2 + S_4.....}$

10 Choose the expansion of $\sinh^{-1}x$.
(i) $\log_e(x - \sqrt{x^2+1})$ (ii) ½ $\log_e\left(\frac{1+x}{1-x}\right)$
(iii) $\log_e(x - \sqrt{x^2-1})$ (iv) $\log_e(x + \sqrt{x^2-1})$

SECTION - B (25 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Show that the sequence $a_n = \frac{n}{n^2+1}$ is decreasing.

OR

b Analyze whether the series $\sum_{n=1}^{\infty} \frac{5}{2n^2+4n+3}$ converges or diverges.

12 a State the series $\sum_{n=1}^{\infty}(-1)^n \frac{n^3}{3^n}$ is absolutely convergent.

OR

b Bring out a power series representation for $\ell_n(1+x)$ and its radius of convergence.

13 a Evaluate $\int_C (2+x^2y)ds$, where C is the upper half of the unit circle $x^2 + y^2 = 1$.

OR

b Analyze whether the vector field $F(x, y) = (x - y)\bar{i} + (x - z)\bar{j}$ is conservative.

14 a Find the flux of the vector field $F(x, y, z) = z\bar{i} + y\bar{j} + x\bar{k}$ across the unit sphere $x^2 + y^2 + z^2 = 1$.

OR

b Evaluate $\int_C F.dr$, where $F(x, y, z) = -y^2\bar{i} + x\bar{j} + z^2\bar{k}$ and C is the curve of intersection of the plane $y + z = 2$ and the cylinder $x^2 + y^2 = 1$.

15 a Expand $\sin^6\theta$ in series of cosines of multiples of θ.

OR

b If $\cos(x + iy) = \cos\theta + i \sin\theta$, prove that $\cos 2x + \cos h\, 2y = 2$.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a (i) Define bounded sequence.

(ii) State and prove monotonic sequence theorem.

OR

b (i) Examine the sum of series $\sum \frac{1}{n^3}$ by using the sum of the first 10 terms. Estimate the error involved in this approximation.

(ii) Discover the number of terms required to ensure that the sum is accurate to within 0.0005.

17 a Test the series $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{n^2}{n^3+1}$ for convergence or divergence.

OR

b Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}$.

18 a Analyze the integral $\int_C y^2 dx + xdy$, where (i) $C = C_1$ is the line segment from (-5, -3) to (0, 2) and (ii) $C = C_2$ is the arc of the parabola $x = 4 - y^2$ from (-5, -3) to (0, 2)

OR

b (i) Show that $F(x, y, z) = y^2z^3\bar{i} + 2xyz^3\bar{j} + 3xy^2z^2\bar{k}$ is a conservative vector field.

(ii) Find a function f such that $F = \nabla f$.

19 a (i) Describe surface area.

(ii) Find the surface area of a sphere of radius a.

OR

b Compute the surface integral $\iint_S x^2 ds$ where S is the unit sphere $x^2 + y^2 + z^2 = 1$.

20 a Express $\frac{\sin 6\theta}{\sin \theta}$ in terms of $\cos\theta$.

OR

b Point out $\sin\phi = \pm\sin^2\alpha = \pm\sinh^2\beta$ if $\cos\alpha\cosh\beta = \cos\phi$ and $\sin\alpha.\sinh\beta = \sin\phi$.

Z-Z-Z END