PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION MAY 2019
(First Semester)

Branch – **MATHEMATICS**

CALCULUS - I

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 The curvature of a circle of radius a is ________.
(i) a^2 (ii) $1/a^2$ (iii) $1/a$ (iv) a^3

2 The plane determined by the unit tangent vector and unit normal vector is called the ______ plane.
(i) Osculating (ii) Normal (iii) Orthogonal (iv) Binormal

3 If $f(x, y) \to L_1$ as $(x, y) \to (a, b)$ along a path C_1 and $f(x, y) \to L_2$ as $(x, y) \to (a, b)$ along a path C_2, where $L_1 \neq L_2$, then $\lim_{(x,y)\to(a,b)} f(x,y)$ ________.
(i) Does not exist (ii) = 0 (iii) is finite (iv) $= L_1$

4 If $f(x, y) = x^3 + x^2y^3 - 2y^2$, then $f_x(2, 1) =$ ________.
(i) 32 (ii) 8 (iii) 16 (iv) 64

5 If $f(x, y) = \sin x + e^{xy}$, then $\nabla f(0, 1) =$ ________.
(i) (0, 2) (ii) (2, 0) (iii) (1, 0) (iv) (0, 1)

6 If the graph of a function f has a tangent plane at a local maximum or minimum, then the tangent plane must be ________.
(i) Perpendicular (ii) Parallel (iii) Vertical (iv) Horizontal

7 The moment of the entire lamina about the x axis is _______.
(i) $\iint_D xe(x,y)dA$ (ii) $\iint_D yp(x,y)dA$ (iii) $\iint_D p(x,y)dA$ (iv) $\iint_D xyp(x,y)dA$

8 The moment of inertia about the origin is also called the ______.
(i) Second moment (ii) First moment
(iii) Polar moment of inertia (iv) Third moment

9 The Jacobian of the transformation T given by $x = r\cos\theta$, $y = r\sin\theta$ is ________.
(i) r (ii) r^2 (iii) $\sin\theta$ (iv) $\cos\theta$

10 A solid region E is said to be of _______ if it lies between the graphs of two continuous functions of x and y.
(i) type I (ii) type II (iii) type III (iv) type IV

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Find the curvature of the parabola $y = x^2$ at the points (0, 0), (1, 1), (2, 4).

OR

b Find the curvature of the twisted cubic $r(t) = (t, t^2, t^3)$ at a general point and at (0, 0, 0).

12 a i) If $Z = f(x, y) = x^2 + 3xy - y^2$, find the differential d^2.
ii) If x changes from 2 to 2.05 and y changes from 3 to 2.96, compare the value of Δz and dz.

OR

b Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point (1, 1, 3).

13 a Find the extreme values of $f(x, y) = y^2 - x^2$.

OR

b If $f(x, y, z) = x \sin yz$, (a) Find the gradient of f and (b) Find the directional derivative of f at (1, 3, 0) in the direction of $v = i + 2j - k$.

Cont...

14 a Find the moments of inertia I_x, I_y and I_o of a homogeneous disk D with density p(x, y) = p, center the origin, and radius a.

OR

b Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ above the xy plane, and inside the cylinder $x^2 + y^2 = 2x$.

15 a Use spherical coordinates to find the volume of the sold that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$.

OR

b A solid E lies within the cylinder $x^2 + y^2 = 1$, below the plane z = 4, and above the paraboloid $z = 1 - x^2 - y^2$. The density of any point is proportional to its distance from the axis of the cylinder. Find the mass of E.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a Find the equations of the normal plane and osculating plane of the helix r(t) = cost i + sint j + tk at the point $p(0, 1, \pi/2)$.

OR

b Prove that the curvature of the curve given by the vector function r is

$$\eta(t) = \frac{|r'(t) \text{ x } r''(t)|}{|r'(t)|^3}.$$

17 a Show that $(x, y)^{\lim} \to (0, 0)\ \frac{x^2 - y^2}{x^2 + y^2}$ does not exist.

OR

b i) Calculate f_{xxyz} if f(x, y, z) = sin(3x + yz).

ii) Show that the function u(x, y) = ex sin y is a solution of Laplace's equation.

18 a Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point (3, 1, -1).

OR

b Find the shortest distance from the point (1, 0, -2) to the plane x + 2y + z = 4.

19 a The manager of a movie theater determines that the average time movie-goers wait in line to buy a ticket for this week's film is 10 minutes and the average time they wait to buy popcorn is 5 minutes. Assuming that the waiting times are independent, find the probability that a movie goes waits a total of less than 20 minutes before taking his or her seat.

OR

b The density at any point on a semicircular lamina is proportional to the distance from the center of the circle. Find the center of mass of the lamina.

20 a Express the iterated integral $\int_0^1 \int_0^{x^2} \int_0^y f(x,y,z)\ dz\ dy\ dx$ as a triple integral and then rewrite it as an iterated integral in a different order, integrating first with respect to x, then z, and then y.

OR

b Evaluate $\iiint_E \sqrt{x^2 + z^2}\ dv$, where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane y = 4.

Z-Z-Z

END