PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION MAY 2019
(Second Semester)

Branch – **MATHEMATICS**

ANALYTICAL GEOMETRY

Time: Three Hours   Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks   (10 x 1 = 10)

1 What is the polar equation of the conic?

(i) $\dfrac{\ell}{r} = 1 - e\cos\theta$ (ii) $\dfrac{\ell}{r} = 1 + e\cos(\theta - \alpha)$ (iii) $\dfrac{1}{r} = 1 + e\cos\theta$ (iv) $\dfrac{\ell}{r} = 1 + e\cos\theta$

2 Find the distance between $(1, \pi)$ and $(2, \pi)$

(i) 7   (ii) 9   (iii) 0   (iv) 5

3 Which is the equation of the following lines by joining the points (2, 5, 8) and (-1, 6, 3)?

(i) $\dfrac{x+2}{3} = \dfrac{y+5}{-1} = \dfrac{z-8}{5}$ (ii) $\dfrac{x+2}{-3} = \dfrac{y-5}{1} = \dfrac{z-8}{-5}$

(iii) $\dfrac{x-2}{-3} = \dfrac{y-5}{1} = \dfrac{z-8}{-5}$ (iv) $\dfrac{x-2}{3} = \dfrac{y-5}{-1} = \dfrac{z-8}{5}$

4 Identify the formula for finding shortest distance between two lines

$$\dfrac{x-x_1}{\ell_1} = \dfrac{y-y_1}{m_1} = \dfrac{z-z_1}{n_1} \text{ and } \dfrac{x-x_2}{\ell_2} = \dfrac{y-y_2}{m_2} = \dfrac{z-z_2}{n_2}$$

(i) $(x_2-x_1)\ell + (y_2-y_1)m + (z_2-z_1)n$ (ii) $(x_2+x_1)\ell + (y_2+y_1)m + (z_2+z_1)n$

(iii) $(x_1+y_1)\ell + (x_2+y_2)m + (z_2-z_1)n$ (iv) $(x_2-x_1)n + (y_2-y_1)m + (z_2-z_1)\ell = 0$

5 Which one of the following represents a sphere equation?

(i) $x^2 + y^2 + z^2 - 2ux - 2vy - 2wz - d^2 = 0$ (ii) $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

(iii) $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d^2 = 0$ (iv) $x^2 + y^2 + z^2 - 2ux - 2vy - 2wz - d = 0$

6 Find the equation of the sphere with centre (0, 0, 0) and radius is 3.

(i) $x^2+y^2+z^2 = 3$ (ii) $x^2+y^2+z^2 = 9$ (iii) $x^2+y^2+z^2 = -3$ (iv) $x^2+y^2+z^2 = -9$

7 Identify the cone equation of the following

(i) $x^2 + y^2 + z^2 + 2fyz + 2gzx + 2hxy = 0$ (ii) $ax^2 + by^2 + cz^2 - 2fyz - 2gzx - 2hxy = 0$

(iii) $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ (iv) $x^2 + y^2 + z^2 - 2fyz - 2gzx - 2hxy = 0$

8 Mention the equation of the tangent plane at (x_1, y_1, z_1) to the cone.

(i) $(x + x_1)(ax_1 + fy_1 + hz_1) + (y + y_1)(gx_1 + by_1 + hz_1) + (z + z_1)(fx_1 + gy_1 + cz_1) = 0$

(ii) $(x + x_1)(ax_1 + hy_1 + gz_1) + (y + y_1)(hx_1 + by_1 + fz_1) + (z + z_1)(gx_1 + fy_1 + cz_1) = 0$

(iii) $(x - x_1)(ax_1 + fy_1 + hz_1) + (y - y_1)(gx_1 + by_1 + hz_1) + (z - z_1)(fx_1 + gy_1 + cz_1) = 0$

(iv) $(x - x_1)(ax_1 + fy_1 + gz_1) + (y - y_1)(hx_1 + by_1 + fz_1) + (z - z_1)(gx_1 + fy_1 + cz_1) = 0$

9 A cylinder is a surface generated by a _____.

(i) plane   (ii) straight line   (iii) parallel line   (iv) sphere

10 Mention the equation of the enveloping cylinder of the surface $ax^2 + by^2 + cz^2 = 1$ having the generator parallel to $\dfrac{x}{\ell} + \dfrac{y}{m} + \dfrac{z}{n}$.

(i) $(ax\ell + bym + czn)^2 = (a\ell^2 + bm^2 + cn^2)(ax^2 + by^2 + cz^2 - 1)$

(ii) $(ax\ell + bym + czn) = (a\ell^2 + bm^2 + cz^2)(ax^2 + by^2 + cn^2 - 1)$

(iii) $(ax\ell + bym + czn) = (a\ell^2 + bm^2 + cn^2)(ax^2 + by^2 + cz^2 - 1)$

(iv) $(ax\ell + bym + czn)^2 = (a\ell^2 + bm^2 + cn^2)^2(ax^2 + by^2 + cz^2 - 1)^2$

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks   (5 x 5 = 25)

11 a Show that in a conic the semi-latus rectum is the harmonic mean between the segments of a focal chord.

OR

b Bring out the equation of the normal at the point P whose vectorial angle is α.

$+ 5y - z - 7 = 0$ and $2x - 5y + 3z = 1$.

OR

b Show that the lines $\dfrac{x-3}{2} = \dfrac{y-2}{-5} = \dfrac{z-1}{3}$ and $\dfrac{x-1}{-4} = y+2 = \dfrac{z-6}{2}$ are coplanar.

13 a Bring out the sphere equation which has its centre at the point (6, -1, 2) and touches the plane $2x - y + 2z - 2 = 0$.

OR

b Produce the equation of the sphere having the circle $x^2 + y^2 + z^2 - 2x + 4y - 6z + 7 = 0$; $2x - y + 2z = 5$ for a great circle.

14 a Bring out the equation of the cone with vertex 0 and base curve the conic in which the surface $ax^2 + by^2 + cz^2 = 1$ is cut by the plane $\ell_1 x + m_1 y + n_1 z = p$.

OR

b Show that the general equation to a cone which touches the co-ordinate planes is $\sqrt{fx} \pm \sqrt{gy} \pm \sqrt{hz} = 0$.

15 a Find the equation of the cylinder whose generators are parallel to the line $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{3}$ and guiding curve $x^2 + y^2 = 16$, $z = 0$.

OR

b Bring out the equation of a right circular cylinder of radius 3 with axis $\dfrac{x+2}{3} = \dfrac{y-4}{6} = \dfrac{z-1}{2}$.

SECTION -C (40 Marks)
Answer ALL questions
ALL questions carry EQUAL Marks   (5 x 8 = 40)

16 a Trace the conic $\dfrac{1}{r} = 1 + e\cos\theta$.

OR

b Discuss the equation of the chord of the conic $\dfrac{\ell}{r} = 1 + e\cos\theta$ joining the points whose vectorial angles are $\alpha - \beta$ and $\alpha + \beta$.

17 a Prove that the lines $\dfrac{x+1}{-3} = \dfrac{y+10}{8} = \dfrac{z-1}{2}$ and $\dfrac{x+3}{-4} = \dfrac{y+1}{7} = \dfrac{z-4}{1}$ are coplanar. Also find their point of intersection and the plane through them.

OR

b Find the shortest distance between the lines $\dfrac{x-3}{-1} = \dfrac{y-4}{2} = \dfrac{z+2}{1}$; $\dfrac{x-1}{1} = \dfrac{y+7}{3} = \dfrac{z+2}{2}$.

18 a Find the equation of the sphere through the four points (2, 3, 1), (5, -1, 2), (4, 3, -1) and (2, 5, 3).

OR

b Examine that the plane $2x - y - 2z = 16$ touches the sphere $x^2 + y^2 + z^2 - 4x + 2y + 2z - 3 = 0$ and find the point of contact.

19 a Find the equation of the cone whose vertex is (1, 2, 3) and which passes through the circle $x^2 + y^2 + z^2 = 4$, $x + y + z = 1$.

OR

b Find the equation of the tangent plane to the cone $9x^2 - 4y^2 + 16z^2 = 0$ which contain the line $\dfrac{x}{32} = \dfrac{y}{72} = \dfrac{z}{27}$.

20 a Find the equation of the cylinder whose generators are parallel to the z axis and the guiding curve is $ax^2 + by^2 = cz$, $\ell x + my + nz = p$.

OR

b Discuss the equation of the right circular cylinder described on the circle passing through the points (a, 0, 0), (0, a, 0), (0, 0, a) as a guiding curve.

Z-Z-Z   END