PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2019
(Sixth Semester)

Branch – **MATHEMATICS**

ALGEBRA - II

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 Define symmetric and skew-symmetric matrices and also give an example.
2 Define an orthogonal matrix and give an example.
3 The set of all real polynomials of degree $n \geq 1$ does not form a vector space over R. Why?
4 Define base.
5 Define inner product space.
6 Prove that $\|\alpha u\| = |\alpha| \|u\|$.
7 Define rank of a matrix.
8 Define characteristic polynomial.
9 If $T \in A(v)$ and if $S \in A(v)$ is regular then $r(T) = r(ST S^{-1})$.
10 Define characteristic root.

SECTION - B (25 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Show that any square matrix is expressible as the sum of a symmetric and a Skew-symmetric matrix in a unique way .

OR

b Show that the matrix $U = \frac{1}{5}\begin{pmatrix} -1+2i & -4-2i \\ 2-4i & -2-i \end{pmatrix}$ is unitary.

12 a If V is the internal direct sum of $U_1, \ldots.. U_n$ then prove that V is isomorphic to the external direct sum of $U_1, \ldots. U_n$.

OR

b If L(S) is a subspace of V.

13 a If $\{v_i\}$ is an orthonormal set, then prove that the vectors in $\{v_i\}$ are linearly independent. If $w = \alpha_1 v_1 + \ldots. + \alpha_n v_n$ then prove that $\alpha_i = (w, v_i)$ for $i = 1, 2, \ldots.n$.

OR

b If $u, v \in \text{v}$ and $\alpha, \beta \in F$ then prove that
$(\alpha u + \beta v, \alpha u + \beta v) = \alpha\bar{\alpha}(u, u) + \alpha\bar{\beta}(u, v) + \bar{\alpha}\beta(v, u) + \beta\bar{\beta}(v, v)$.

14 a Determine the characteristic roots of the matrix $A = \begin{pmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{pmatrix}$.

OR

b Prove that the characteristic root of a hermitian matrix are all real.

Cont ...

15 a If v is finite dimensional over F and if $T \in A(V)$ is singular then prove that there exits an $S \neq 0$ in A(v) such that $ST = TS = 0$.

OR

b Prove that an element $\lambda \in F$ is a characteristic root of $T \in A(V)$ if and only if for some $v \neq 0$ in v, $vT = \lambda v$.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Let A and B be complex matrices such that the product AB is defined. Then prove that

(i) $\overline{AB} = \overline{A}\ \overline{B}$ (ii) $(AB)^* = B^*A^*$

(iii) $(\overline{A})^{-1} = \overline{(A^{-1})}$ (iv) $(A^*)^{-1} = (A^{-1})^*$

17 If $v_1, \ldots, v_n \in v$ are linearly independent then prove that every element in linear span has unique representation in the form $\lambda_1 v_1 + \ldots + \lambda_n v_n$ with the $\lambda_i \in F$.

18 If V is finite dimensional and $v \neq 0 \in v$ then prove that there is an element $f \in V$ such that $f(v) \neq 0$.

19 Verify the Cayley – Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{pmatrix}$ and hence find its inverse.

20 If V is finite dimensional over F then for $S, T \in A(V)$ prove that

i) $r(ST) \leq r(T)$ ii) $r(TS) \leq r(T)$

iii) $r(ST) = r(TS) = r(T)$ for S regular in A(V).

Z-Z-Z END