PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2019
(Fifth Semester)

Branch – **MATHEMATICS**

REAL ANALYSIS

Time : Three Hours   Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks   (10 x 2 = 20)

1 Define an equivalence relation.

2 Prove that every neighborhood is an open set.

3 Define open cover and compact set.

4 Define separated and connected.

5 Define convergent sequence.

6 Define Cauchy sequence.

7 Define continuous function.

8 Write the function of which is continuous at x = 0 and has a discontinuity of the second kind.

9 Define local maximum.

10 Find the derivative of $f(x) = \begin{cases} x^2 \sin\frac{1}{x} & (x \neq 0) \\ 0 & (x = 0) \end{cases}$.

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Define countable and uncountable sets and prove that every infinite subset of a countable set A is countable.

OR

b Prove that a set E is open if and only if its complement is closed.

12 a Suppose $K \subset Y \subset X$. Then prove that K is compact relative to X if and only if K is compact relative to Y.

OR

b Prove that a subset E of the real line R is connected if and only if it has the following property. If $x \in E$, $y \in E$, and $x < z < y$, then $z \in E$.

13 a Prove that the sub sequential limits of a sequence $\{p_n\}$ in a metric space x form a closed subset of x.

OR

b Suppose $\{S_n\}$ is monotonic. Then prove that $\{S_n\}$ converges if and only if it is bounded.

14 a Suppose f is a continuous mapping of a compact metric space X into metric space Y. then prove that f(x) is compact.

OR

b Prove that if f is a continuous mapping of a metric space x into a metric space Y, and if E is a connected subset of x, then f(E) is connected.

15 a Let f be defined on [a, b]. Prove that if f has a local maximum at a point $x \in (a, b)$, and if $f'(x)$ exists, then $f'(x) = 0$.

OR

b Suppose f is a real differentiable function on [a, b] and suppose $f'(a) < \lambda < f'(b)$. Then prove that there is a point $x \in (a, b)$ such that $f'(x) = \lambda$.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 a) Let $\{E\alpha\}$ be a (finite or infinite) collection of sets $E\alpha$. Then prove that

$$\left(\bigcup_{\alpha} E_{\alpha}\right)^{c} = \bigcap_{\alpha}\left(E_{\alpha}^{c}\right)$$

b) Suppose $Y \subset X$. Prove that a subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X.

17 Prove that every k-cell is compact.

18 Prove that for any sequence $\{C_n\}$ of positive numbers,

$$\liminf_{n \to \infty} \frac{C_{n+1}}{C_n} \le \liminf_{n \to \infty} \sqrt[n]{C_n},$$

$$\limsup_{n \to \infty} \sqrt[n]{C_n}, \le \limsup_{n \to \infty} \frac{C_{n+1}}{C_n}$$

19 Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove that f is uniformly continuous on X.

20 State and prove the generalized mean value theorem.

Z-Z-Z END