14 a Show that the transform of $e^{-\frac{x^2}{2}}$ is $e^{-\frac{s^2}{2}}$ by finding the Fourier transform of $e^{-a^2x^2}$, $a > 0$.

OR

b Find Fourier cosine transform of $f(x) = \begin{cases} \cos x & \text{in } 0 < x < a \\ 0 & x \geq a \end{cases}$.

15 a Solve $\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2}, -\infty < x < \infty, t \geq 0$ with conditions $u(x, 0) = f(x)$, $\frac{\partial u}{\partial t}(x,0) = g(x)$ and assuming $u, \frac{\partial u}{\partial x} \to 0$ as $x \to \pm\infty$.

OR

b Find f(x) if its finite Fourier cosine transform is $\frac{2\iota^3}{p^2\pi^2}(-1)^p$ for $p = 1, 2, 3....$ and is $\frac{\iota^3}{3}$ for $p = 0 : 0 < x < \iota$.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 i) Solve $z = p^2 + q^2$

ii) Solve $p - x^2 = q + y^2$.

17 Solve $z^2 = pqxy$ by charpit's method.

18 Obtain Fourier series for the function $f(x) = x \sin x$ in $0 < x < 2\pi$.

19 Find the Fourier transform of f(x) given by

$$f(x) = \begin{cases} 1 & \text{for } |x| < a \\ 0 & \text{for } |x| > a > 0 \end{cases}$$

and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$ and $\int_{-\infty}^{\infty} \frac{\sin as \cos sx}{s} ds$.

20 Find the finite Fourier sine transform of $f(x) = 1$ in $(0, \pi) = 1$ in $(0, \pi)$. Use the inversion theorem and find Fourier sine series for $f(x) = 1$ in $(0, \pi)$ Hence prove

(i) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ... = \pi/4$ (ii) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... = \pi^2/8$.

Z-Z-Z END

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