PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2019
(Sixth Semester)

Branch – **MATHEMATICS**

GRAPH THEORY

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 What is simple graph?
2 Define regular graph.
3 What you mean pendant vertices in a tree?
4 Define rooted trees.
5 What is planar graph?
6 Define infinite region.
7 Define binary matrix.
8 Write any two property of path matrix.
9 Define complete diagraphs.
10 Define relation matrices.

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Prove that the number of vertices of odd degree in a graph is always even.

OR

b Define walks, paths and circuits with examples.

12 a Prove that a tree with n vertices has n-1 edges.

OR

b Prove that every connected graphs has atleast one spanning tree.

13 a Write the properties of Kuratowski graph.

OR

b Prove for any simple connected planar graph with f regions, n vertices and e edges, $e \geq \frac{3}{2}f$ and $e \leq 3n - 6$.

14 a Prove that the reduced incidence matrix of a tree is non-singular.

OR

b Write the relationships among A_f, B_f and C_f with usual notations.

15 a Define circuit matrix of a diagraph with an example.

OR

b Define (i) Simple digraphs (ii) Asymmetric digraphs and (iii) Symmetric digraphs.

SECTION - C (30 Marks)
Answer any **THREE** Questions
ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Prove that a simple graph with n vertices and k components can have atmost (n-k) (n-k+1)/2 edges.

17 Prove that every tree has either one or two centres.

18 Prove that a connected planae graph with n vertices and e edges has e-n+2 regions.

19 Prove that if B is a circuit matrix of a connected graph G with e edges and n vertices then rank of B = e-n+1.

20 Prove that an arborescence is a tree in which ever vertex other than the root has an in-degree of exactly one.