# PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

**BSc DEGREE EXAMINATION MAY 2019**
(Fifth Semester)

Branch – **MATHEMATICS**

## ALGEBRA - I

Time : Three Hours                                                Maximum : 75 Marks

**SECTION-A (20 Marks)**
Answer **ALL** questions
**ALL** questions carry **EQUAL** marks          (10 x 2 = 20)

1 Define a mapping with an example.

2 Define subgroup of G and right coset of H in G.

3 Define the subgroup HK.

4 Define normal subgroup.

5 Define an automorphism of a group.

6 Define even permutation and if S has 9 elements and find the value of (1, 2, 3) (5, 6, 4, 1, 8).

7 Define zero-divisor and an integral domain.

8 Define homomorphism and Kernel of a homomorphism.

9 Define Maximal ideal of R.

10 Prove that a Euclidean ring possesses a unit elt.

**SECTION - B (25 Marks)**
Answer **ALL** Questions
**ALL** Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a If G is a group, then prove the following (i) The identity elt of G is unique (ii) Every $a \in G$ has a unique inverse in G (iii) For every $a \in G$, $(a^{-1})^{-1} = a$.

OR

b Define the order of an element $a \in G$ and prove that if G is a finite group and $a \in G$, then 0 (a) / 0 (G).

12 a Prove that HK is a subgroup of G if and only if HK = KH.

OR

b Prove that N is a normal subgroup of G if and only if $gNg^{-1} = N$ for every $g \in G$.

13 a Prove that if G is a group then A(G), the set of auto morphisms of G, is also a group.

OR

b Prove that every permutation is the product of its cycles.

14 a Prove that a finite integral domain is a field.

OR

b If $\phi$ is a homomorphism of R into 'R' with Kernel $I(\phi)$, then prove that (i) $I\ \phi$ is a subgroup of R under addition (ii) If $a \in I(\phi)$ and $r \in R$ then both ar and ra are in $I(\phi)$.

**Cont ...**

15 a Prove that if R is a commutative ring with unit element and M is an ideal of R, then M is a maximal ideal of R if and only if $\frac{R}{M}$ is a field.

OR

b Let R be a Euclidean ring and a, b $\in$ R. If $b \neq 0$ is not a unit in R, then prove that $d(a) < d(ab)$.

## SECTION - C (30 Marks)

Answer any **THREE** Questions

**ALL** Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Let G be the set of all 2 x 2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c, d are real numbers such that $ad - bc \neq 0$. Prove that G is an infinite, non-abelian group with respect multiplication.

17 Prove that if $\phi$ is a homomorphism of G into $\overline{G}$ with Kernel K, then K is a normal subgroup of G.

18 State and prove Cayley's theorem.

19 Prove that if U is an ideal of the ring R, then $\frac{R}{U}$ is a ring and is a homomorphic image of R.

20 i) Let R be a Euclidean ring. Then prove that any two elements a and b in R have a greatest common divisor d. Also prove that $d = \lambda a + \mu b$ for some $\lambda, \mu \in R$.

ii) Let R be a Euclidean ring. Then prove that every element in R is either a unit in R or can be written as the product of a finite number of prime elements of R.

Z-Z-Z END