

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)BSc DEGREE EXAMINATION DECEMBER 2019
(Second Semester)

Branch – STATISTICS

PROBABILITY & DISTRIBUTIONS - I

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 x 1 = 10)

- 1 If ACB then
(i) $P(A) \leq P(B)$ (ii) $P(A) = P(B)$
(iii) $P(A) \neq P(B)$ (iv) $P(A) \geq P(B)$
- 2 An integer is chosen at random out of the integers from 1 to 100. What is the probability that it is greater than 70?
(i) $\frac{35}{100}$ (ii) $\frac{30}{100}$
(iii) $\frac{32}{100}$ (iv) $\frac{38}{100}$
- 3 If a random variable x denotes the number of heads obtained in the two tosses. Then the range space of x is
(i) $\{0\}$ (ii) $\{0, 1\}$
(iii) $\{2\}$ (iv) $\{0, 1, 2\}$
- 4 If c is a constant then $E(c)$ is
(i) C^2 (ii) C^3
(iii) C^4 (iv) C
- 5 The distribution function of the two-dimensional random variable (x, y) is a real valued function F defined for all real x and y by the relation
(i) $F(x, y) = P[X=x, Y=y]$ (ii) $F(x, y) = P(x)$
(iii) $F(x, y) = P(y)$ (iv) $F(x, y) = P(X \leq x, Y \leq y)$
- 6 If F is a distribution function of a two-dimensional random variable then $F(-\infty, +\infty)$ is equal to
(i) -1 (ii) $+1$
(iii) $-\infty$ (iv) $+\infty$
- 7 $M_{cx}(t)$ is equal to _____ where 'c' is a constant
(i) $M_x(t)$ (ii) $M_c(xt)$
(iii) $M_x(ct)$ (iv) $M_t(x)$
- 8 Cumulants generating function $k(t)$ is defined as
(i) $M_x(t)$ (ii) $M_t(x)$
(iii) $\log M_t(x)$ (iv) $\log M_x(t)$
- 9 If the random variables u and v be transformed to the random variables x and y , then the Jacobian of the transformation is defined as
(i) $J = \frac{\partial(x, y)}{\partial(u, v)}$ (ii) $J = \frac{\partial x}{\partial u}$
(iii) $J = \frac{\partial y}{\partial u}$ (iv) $J = \frac{\partial^2(x, y)}{\partial^2(u, v)}$

- 10 If the cumulative distribution function of a continuous random variable X is $F(x)$, then the cumulative distribution function of $y = x + a$ is
- (i) $F(x + a)$ (ii) $F(x - a)$
 (iii) $F(x)$ (iv) $aF(x)$

SECTION - B (25 Marks)Answer **ALL** questions**ALL** questions carry **EQUAL** Marks (5 x 5 = 25)

- 11 a State and prove multiplication theorem on probability.
OR
b Define (i) Classical (ii) Statistical and (iii) Axiomatic probability.
- 12 a A random variable x has the following probability function
- | | | | | | | | | | |
|--------|---|---|-----|------|------|------|-------|--------|----------|
| x | : | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $p(x)$ | : | 0 | k | $2k$ | $2k$ | $3k$ | k^2 | $2k^2$ | $7k^2+k$ |
- Find (i) k (ii) $p(x < 3)$.
OR
b i) Define distribution function (ii) State its properties (iii) Prove any one of the properties of the distribution function.
- 13 a Write short notes on Bi-variate distributions.
OR
b Test whether x and y are independent variables given that $f(x, y) =$
- $$\begin{cases} 4xy & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$
- 14 a Prove that sum of two independent random variables is equal to the product of their characteristic functions.
OR
b Write short notes on convergence in probability.
- 15 a Let (x, y) be a two dimensional continuous random variable with $f(x, y) = 8xy, 0 < y < x < 1$. Find $E[y/x]$.
OR
b If x and y are independent, prove that $E[y/x] = E[y]$.

SECTION -C (40 Marks)Answer **ALL** questions**ALL** questions carry **EQUAL** Marks (5 x 8 = 40)

- 16 a State and prove Baye's theorem.
OR
b Three urns of the same appearance have the following proportion of balls. First urn: 2 black, 1 white; Second urn; 1 black, 2 white; Third urn : 2 black 2 white. One of the urns is selected and one ball is drawn. It turns out to be white what is the probability of drawing a white ball again, the first one not having been returned?

- 17 a A coin is tossed until a head appears. What is the expectation of the number of losses?

OR

- b Given the following bivariate probability distribution. Obtain (i) Marginal distribution (ii) Conditional distribution of x when y = 2.

↓y	-1	0	1
x→			
0	1/15	2/15	1/15
1	3/15	2/15	1/15
2	2/15	1/15	2/15

- 18 a State and prove Chebyshev's inequality.

OR

- b State and prove Bernoulli's weak law of large numbers.

- 19 a For the joint distribution

$$f(x, y) = \begin{cases} \frac{9}{4} - x - y & 0 \leq x \leq 2, 0 \leq y \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

obtain the marginal and conditional distribution of x given y.

OR

- b Given the joint p.d.f. of (x,y) as $f(x, y) = \begin{cases} \frac{k}{(1+x+y)^3} & x > 0, y > 0 \\ 0 & \text{elsewhere} \end{cases}$

Find (i) k (ii) marginal density of x.

- 20 a Explain in detail conditional expectation and conditional variance with an example.

OR

- b Discuss in detail on transformation of random variables.

Z-Z-Z

END