### PSG COLLEGE OF ARTS & SCIENCE (AUTONOMOUS)

# **BSc DEGREE EXAMINATION DECEMBER 2019**

(First Semester)

#### Branch - STATISTICS

### **MATRICES**

Time: Three Hours

Maximum: 75 Marks

### SECTION-A (20 Marks)

Answer ALL questions

**ALL** questions carry **EQUAL** marks  $(10 \times 2 = 20)$ 

- 1 Define Hermitian matrix.
- 2 Define Transpose of a matrix.

3 If 
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$
 the  $|A| = ?$ 

- 4 Define inverse of a matrix.
- 5 Define rank of a matrix.
- Find the rank of  $A = \begin{pmatrix} 1 & 4 & 2 \\ 2 & 0 & 1 \end{pmatrix}$ 6
- 7 Define characteristic equation of a matrix.
- 8 State Cayley-Hamilton theorem.
- 9 Define vector space.
- 10 Define matrix of quadratic form.

## SECTION - B (25 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks  $(5 \times 5 = 25)$ 

11 a Show that, a necessary and sufficient condition for a square matrix A to be symmetric is that A<sup>t</sup>=A.

- b Show that, each diagonal element of a Hermitian matrix is purely real.
- 12 a Show that the value of a determinant remains unaltered if its rows and columns are interchanged.

OR

- Find the inverse of the matrix  $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$ . b
- Find the rank of  $A = \begin{pmatrix} 0 & i & -i \\ -i & 0 & i \\ i & -i & 0 \end{pmatrix}$ .
  - Does the following system of equations possess a common non-zero solution? 2x-3y+z=0x + 2y - 3z = 04x-y+2z=0
- Obtain the characteristic equation and characteristic roots of the matrix 14 a

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

Show that the characteristic roots of a real symmetric matrix are real.

Cont...

Prove that the intersection of two sunspaces of a vector space is a subspace.

OR

b Obtain the matrix of the quadratic form  $3x_1^2 + x_1x_2 - 4x_2^2$  in  $x_1$  and  $x_2$ .

# SECTION - C (30 Marks)

Answer any THREE Questions

ALL Questions Carry EQUAL Marks (3 x 10 = 30)

Verify that the matrix 
$$\frac{1}{3}\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{pmatrix}$$
 is orthogonal.

17 Solve the following equations with the help of determinants.

$$5x-6y+4z=15$$

$$7x+4y-3z=19$$

$$2x+y+6z=46$$

18 Find the rank of a matrix 
$$A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{pmatrix}$$

Verify Cayley-Hamilton theorem for the matrix 
$$A = \begin{pmatrix} -3 & 5 & 1 \\ 2 & 0 & -1 \\ 1 & -2 & 3 \end{pmatrix}$$

Show that, in  $V_3(R)$  the vectors (1,2,1), (2,1,0) and (1,-1,2) are linearly independent.