PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION DECEMBER 2019
(First Semester)

Branch – **STATISTICS**

MATHEMATICS – I

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 If an equation f(x)=0 has degree n then the equation has ___ number of roots.
(i) n-1 (ii) n (iii) n+1 (iv) 0

2 If α,β,γ are the roots of $x^3+px^2+qx+r=0$ then $\alpha+\beta+\gamma=$_____.
(i) p (ii) q (iii) -p (iv) -q

3 If each diagonal element of a scalar matrix is 1 then the matrix is called
(i) unit matrix (ii) scalar matrix
(iii) zero matrix (iv) non zero matrix

4 A square matrix A is said to be non-singular if
(i) $|A|=\text{adj } A$ (ii) $|A|=-|A|$ (iii) $|A|\neq 0$ (iv) $|A|=0$

5 If $y=\cos x$, then dy/dx is
(i) cos x (ii) -cos x (iii) -sin x (iv) sin x

6 $\cosh^2 x-\sinh^2 x$ is equal to
(i) -1 (ii) 1 (iii) 0 (iv) 2

7 The n^{th} derivative of sin(ax+b) is
(i) $a^n\sin\left(\frac{n\pi}{2}+ax+b\right)$ (ii) $\sin\left(\frac{n\pi}{2}+ax+b\right)$ (iii) $a^n\cos\left(\frac{n\pi}{2}+ax+b\right)$ (iv) none of these

8 If f(x,y) is a homogeneus function of degree n then
(i) $\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}=f$ (ii) $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=xf$ (iii) $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=0$ (iv) $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=1$

9 $\int\frac{dx}{\sqrt{1-x^2}}$ is _____.
(i) $\sin^{-1}x+c$ (ii) $-\sin^{-1}x+c$ (iii) $\cos^{-1}x+c$ (iv) $-\cos^{-1}x+c$

10 If f(-x)=f(x) then f(x) is an ____ function.
(i) odd (ii) even (iii) inverse (iv) identity

SECTION - B (25 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Solve the equation $x^4-6x^3+11x^2-10x+2=0$, given that $2+\sqrt{3}$ is a root of the equation.

OR

b If α,β,γ are the roots of the equation $x^3+px^2+qx+r=0$, form the equation whose roots are $\alpha\beta,\beta\gamma$ and $\gamma\alpha$.

12 a Show that $A=\begin{bmatrix}\frac{1}{3} & \frac{2}{3} & \frac{2}{3}\\ \frac{2}{3} & \frac{1}{3} & \frac{-2}{3}\\ \frac{-2}{3} & \frac{1}{3} & \frac{-1}{3}\end{bmatrix}$ is orthogonal.

OR

b Find the rank of $\begin{bmatrix}1 & 2 & 3\\ -1 & -2 & -3\\ 2 & 4 & 6\end{bmatrix}$

13 a Find $\frac{dy}{dx}$, when $y=x(x^2-1)(x^2+4)$.

OR

b If $y=\frac{\sqrt{x}}{2x+3}$, then find $\frac{dy}{dx}$.

14 a Find y_n where $y=\frac{3}{(x+1)(2x-1)}$.

OR

b If $V=(x^2+y^2+z^2)^{-1/2}$ show that $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}=0$.

15 a Calculate $\int \tan\theta d\theta$.

OR

b Prove that $\int_0^{\pi/2} \frac{(\sin x)^{3/2}}{(\sin x)^{3/2}+(\cos x)^{3/2}}dx=\frac{\pi}{4}$.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a Solve $x^4-8x^3+14x^2-8x-15=0$, given that the sum of roots is equal to the sum of the other two.

OR

b Solve the equation $2x^5-15x^4+37x^3-37x^2+15x-2=0$.

17 a Show that the following equations:
$2x-y+z=7$; $3x+y-5z=13$; $x+y+z=5$ are consistent and solve them.

OR

b Find the characteristic equation of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence obtain its inverse.

18 a Differentiate $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$.

OR

b If $x(1+y)^{1/2}+y(1+x)^{1/2}=0$ then prove $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$.

19 a If $y=\sin(m \sin^{-1} x)$, then show that $(1-x^2)y_2-xy_1+m^2y=0$ and $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+(m^2-n^2)y_n=0$.

OR

b If $z=f(x,y)$ and $x=r\cos\theta$; $y=r\sin\theta$ prove that $\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2=\left(\frac{\partial z}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$.

20 a Evaluate $\int \frac{2dx}{(1-x)(1+x^2)}$

OR

b Evaluate $\int \frac{x+\sin x}{1+\cos x}dx$.

Z-Z-Z END