PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION DECEMBER 2019
(First Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

CALCULUS

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 The parametric equations for the curve are x = cost, y = sint, z = 1. Then the curve is
(i) twisted cubic (ii) helix (iii) toroidal spiral (iv) trefoil knot

2 If $r(t) = 2\cos t\vec{i} + \sin t\vec{j} + 2t\vec{k}$, then the value of $\int_0^{\pi/2} r(t)\,dt =$
(i) $2\vec{i}+\vec{j}+\frac{\pi^2}{2}\vec{k}$ (ii) $2\vec{i}+\vec{j}-\frac{\pi^2}{4}\vec{k}$ (iii) $2\vec{i}+\vec{j}+\frac{\pi^2}{4}\vec{k}$ (iv) $2\vec{i}-\vec{j}+\frac{\pi^2}{4}\vec{k}$

3 The equation $z = \sqrt{9 - x^2 - y^2}$ represents the _____.
(i) sphere (ii) top half of the sphere (iii) circle (iv) top half of the circle

4 The value of $\lim_{(x,y)\to(1,2)} (x^2y^2 - x^3y^2 + 3x + 2y)$ is ________.
(i) 9 (ii) 15 (iii) 11 (iv) 18

5 If $F(x,y)=x^3 + y^3 - 6xy$, then the value of $\frac{dy}{dx} =$
(i) $-\left(\frac{x^2-2y}{y^2-2x}\right)$ (ii) $\left(\frac{x^2-2y}{y^2-2x}\right)$ (iii) $\left(\frac{2y-x^2}{y-2x}\right)$ (iv) $\left(\frac{x-2y}{y-2x}\right)$

6 If $f(x,y)=\sin x + e^{xy}$, then $\nabla f(x,y) = \langle f_x,f_y\rangle = \nabla f(0,1) =$
(i) <2,2> (ii) <0,2> (iii) <2,-2> (iv) <2,0>

7 If f is continuous on a polar rectangle R by change the polar coordinates in a double integral given by $0 \le a \le r \le b$, $\alpha \le \theta \le \beta$, were $0 \le \beta-\alpha \le 2\pi$ then $\iint_R f(x,y)dA =$ ___.
(i) $\int_\alpha^\beta\int_a^b f(r\cos\theta, r\sin\theta)drds$ (ii) $\int_\alpha^\beta\int_a^b f(\cos\theta,\sin\theta)rdrds$
(iii) $\int_\alpha^\beta\int_a^b f(r\sin\theta, r\cos\theta)rdrds$ (iv) $\int_\alpha^\beta\int_a^b f(r\cos\theta, r\sin\theta)rdrds$

8 If x is a random variable with probability density function f, then its mean μ is
(i) $\int_0^\infty xf(x)dx$ (ii) $\int_{-\infty}^\infty f(x)dx$ (iii) $\int_{-\infty}^\infty xf(x)dx$ (iv) $\int_0^\pi xf(x)dx$

9 The joint density function satisfies $f(x,y,z)\ge 0$, and $\int_{-\infty}^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty f(x,y,z)dzdydx =$
(i) 1 (ii) 0 (iii) ∞ (iv) $-\infty$

10 The point having rectangular coordinates whose cylindrical point is $\left(2,\frac{2\pi}{3},1\right)$, is
(i) $(1,\sqrt{3},1)$ (ii) $(-1,\sqrt{3},1)$ (iii) $(1,-\sqrt{3},1)$ (iv) $(1,\sqrt{3},-1)$

SECTION - B (25 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane y + z = 2.

OR

12 a Sketch the level curves of the function f(x,y)=6-3x-2y for the values k = 6,0,6,12.

OR

b Find the second partial derivatives of $f(x,y)=x^3 + x^2y^3 - 2y^2$.

13 a If $u = x^4y+y^2z^3$, where x=rse, $y = rs^{2-t}$ and $z = r^2s \sin t$, find the value of $\frac{\partial u}{\partial r}$ when r = 2, s = 1, t = 0.

OR

b Find the local maximum value, minimum values and saddle points of $f(x,y)=x^4+y^4-4xy+1$.

14 a Evaluate $\iint_0 (x+2y)dxdy$, were D is the region bounded by the parabolas $y=2x^2$ and $y=1+x^2$.

OR

b Find the surface area of the part of the surface $z = x^2+2y$ that lies above the triangular region T in the xy-plane with vertices (0,),(1,0) and (1,1).

15 a Use a triple integral to find the volume of the tetrahedron T bounded by the planes x+2y+z = 2, x = 2y, x = 0 and z = 0.

OR

b The point $\left(0, 2\sqrt{3}, -2\right)$ is given in rectangular coordinates. Find spherical coordinates for this point.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a (i) Find the parametric equations for the tangent line to the helix with parametric equations x=2 cos t, y = sin t, z=t at the point $\left(0,1,\frac{\pi}{2}\right)$

(ii) Find the length of the arc of the circular helix with vector equation $r(t) = \cos t\vec{i} + \sin t\vec{j} + t\vec{k}$ from the point (1,0,0) to the point (1,0,2π).

OR

b (i) Show that the curvature of a circle of radius a is $\frac{1}{a}$.

(ii) Find the curvature of the twisted cubic $r(t) = \langle t,t^2,t^3\rangle$ at a general point and at (0,0,0).

17 a (i) If $f(x,y) = \frac{xy}{(x^2+y^2)}$, does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist?

(ii) Find $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+y^2}$ if it exists.

OR

b Show that $f(x,y) = xe^{xy}$ is differentiable at (1,0) and find its linearization there. Using it approximate f(1.1,-0.1)

18 a Find the equations of the tangent plane and normal line at the point (-2,1,-3) to the ellipsoid $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$.

OR

b Find the points on the sphere $x^2+y^2+z^2=4$ that are closest to an farthest from the point (3,1,-1).

19 a Find the volume of the solid that lies under the parabolioid $z = x^2+y^2$ and above the region D in the xy-plane bounded by the line y=2x and the parabola $y=x^2$.

OR

b Find the moments of inertia I_x,I_x, and I_o of a homogenous disk D with density $\rho(x,y) = \rho$, center the origin, and radius a.

20 a Evaluate $\iiint_E \sqrt{x^2+z^2}\, dv$, where E is the region bounded by the paraboloid $y = x^2+z^2$ and the plane y = 4.

OR

b Use spherical coordinates to find the volume of the solid that lies above the cone