14MCU12

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION DECEMBER 2019
(Fourth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ANALYTICAL GEOMETRY OF 3D AND VECTOR CALCULUS

Time : Three Hours   Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks $(10 \times 2 = 20)$

1   Sow that the points $(2,1,1)$ and $(2,5,-1)$ lie on different rids of the plane $x-2y+3z+4=0$.

2   Find the angle between the planes $3x-4y+5z=0$; $2x-y-2z=5$.

3   Find the symmetrical form of line $x+5y-z=7$; $2x-5y+3z+1=0$.

4   Examine the nature of intersection of the planes $2x-5y+z=3$; $x+y+4z=5$; $x+3y+6z=1$.

5   Obtain the equations of the sphere described n the join of points $(2,-3,4)$, $(-5,6,7)$ as diameter.

6   Find the two tangent planes to the sphere $x^2+y^2+z^2-4x+2y+5=0$ which are parallel to the plane.

7   Define rolenoidal and rotational vector.

8   Find the magnitude and direction of the greatest directional derivative of x^2yz^3 at $(2,1,-1)$.

9   Evaluate $\int A.dr$ if $A=x^2i+y^3j$ along $y=x^2$ in the xy plane from $(0,0)$ to $(1,1)$.

10 Using Stoke's theorem, prove that curl grad $\phi=0$.

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks $(5 \times 5 = 25)$

11 a   Find the equation of the plane passing through the points $(3,-8,2)$ and $(-2,-1,1)$ and perpendicular to the plane $x+y+z+1=0$.

OR

b   The vertices of a tetrahedron are $(0,1,2)$, $(3,0,1)$, $(4,3,6)$, $(2,3,2)$. Show that its volume is 6.

12 a   Find the image of the point $(1,3,4)$ under reflection in the plane $2x-y+z+3=0$.

OR

b   Show that the lines $\dfrac{x+5}{3} = \dfrac{y+4}{1} = \dfrac{z-7}{-2}$, $3x+2y+z-2=0=x-3y+2z-3$ are coplanar and find the equation s to the plane in which they lie.

13 a   Find the equation of the sphere through the points $(0,0,0)$, $(0,1,-1)$, $(-1,2,0)$ and $(1,2,3)$.

OR

b   Find the equation of the spheres that passes through the two points $(0,3,0)$, $(-2,-1,-4)$ and cuts orthogonally the two spheres $s:x^2+y^2+z^2+x-3x-2=0$, $s_1:2(x^2+y^2+z^2)+x+3y+4=0$.

14 a   If $\nabla\phi=2xyz^3i+x^2z^3j+3x^2yz^2k$ then find $\phi(x,y,z)$ if $\phi(1,-2,2)=4$.

OR

b   Prove $F=(2xy+z^3)i+x^2j+3xz^2k$ is a conservative force. Find ϕ so that $\nabla\phi=F$.

15 a   Find the work done by the force F=3xyi-5zj+10xk along the curve c, $x=t^2+1$, $y=2t^2$, $z>t^3$ from t=1 to 2.

OR

   b   Find the common area between $y^2=4x$ and $x^2=4y$ by using Green's theorem.

SECTION - C (30 Marks)
Answer any THREE Questions
ALL Questions Carry EQUAL Marks (3 x 10 = 30)

16   The plane x+35y-0z-156=0 is a bisector of the angle between two planes one of which is 4x-3y+12z+13=0. Find the equation of other plane.

17   Find the magnitude ad the equations of the line of shortest distance between the lines $\dfrac{x-8}{3}=\dfrac{y+9}{-6}=\dfrac{z-10}{7}$ and $\dfrac{x-15}{3}=\dfrac{y-29}{8}=\dfrac{z-5}{-5}$.

18   Prove that the plane x+2y-z=4 cuts the sphere $x^2+y^2+z^2-x+z-2=0$ in a circle of radius unity and find the equation of sphere which has this circle for one of its great circle.

19 (i) Find the value of a if (x+3y)i+(y-2z)j+(x+az)k is solenoidal.

   (ii) Prove that div $(r^n r)=(n+3)r^n$ and curl$(r^n r)=0$.

20   Verify Gauss divergence theorem for the function $2xzi+yzj+z^2k$ over the upper half of the sphere $x^2+y^2+z^2=a^2$.

Z-Z-Z   END