PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION DECEMBER 2019
(First Semester)

Branch – **MATHEMATICS**

DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORM

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 The solution of the initial problem $\frac{dy}{dx} = 2x + 3$, $y(1)=2$ is _____.

(i) x^2+3x+2 (ii) x^2+3x-3 (iii) x^2+3x+6 (iv) x^2+3x-6

2 The necessary condition for the differential equation $Mdx+Ndy=0$ to be exact is

(i) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ (ii) $\frac{\partial M}{\partial x} = -\frac{\partial N}{\partial y}$ (iii) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ (iv) $\frac{\partial M}{\partial y} = -\frac{\partial N}{\partial x}$

3 If y_1 and y_2 are linearly dependent solutions of $y'' + p(x)y' + q(x)y = 0$, the value of $W(y_1,y_2)$ is ___.

(i) 0 (ii) 1 (iii) p(x) (iv) q(x)

4 The general solution of $4y'' + 8y^1 + 3y = 0$ is

(i) $c_1e^{x/2} + c_2e^{3x/2}$ (ii) $c_1e^{-x/2} + c_2e^{3x/2}$

(iii) $c_1e^{x/2} + c_2e^{-3x/2}$ (iv) $c_1e^{-x/2} + c_2e^{-3x/2}$

5 The particular solution of the differential equation $y^{11} - y = e^{2x}$ is

(i) $3e^{2x}$ (ii) $\frac{1}{4}e^{2x}$ (iii) $\frac{1}{2}e^{2x}$ (iv) $\frac{1}{3}e^{2x}$

6 The relation between the current I, electromotive force E and the resistance R by Ohm's Law is _____.

(i) $R = I/E$ (ii) $I = E/R$ (iii) $I = R/E$ (iv) $E=I^2R$

7 If $s>0$, then $L[t^n]$=_____$[n\geq 0]$.

(i) $\frac{1}{s^{n+1}}$ (ii) $\frac{n!}{s^n}$ (iii) $\frac{n!}{s^{n+1}}$ (iv) $\frac{1}{s^n}$

8 If $s>0$, then $L^{-1}\left[\frac{K}{(s^2+k^2)}\right]$ = ____.

(i) sin hkt (ii) cos hkt (iii) sin kt (iv) cos kt

9 If $f(t)=g(t)=e^{at}$, then the convolution $f(t)*g(t)$ is _____.

(i) te^{at} (ii) e^{-at} (iii) e^{at} (iv) $-te^{at}$

10 $L^{-1}\left[\frac{e^{-3s}}{s^2}\right]$ is ____.

(i) u(t-3).(t+3) (ii) u(t-3).(t-3) (iii) u(t+3).(t+3) (iv) u(t+3).(t-3)

SECTION - B (25 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Solve the initial value problem $\frac{dy}{dx} = 2x + 1; y(0) = 3$

OR

b Solve the differential equation $\frac{dy}{dx} = (x + y + 3)^2$.

12 a Verify that the functions $y_1(x)=e^x$ and $y_2(x)=xe^x$ are solutions of $y''-2y^1+y=0$ and also find a solution satisfying the initial conditions $y(0)=3$, $y^1(0)=1$.

OR

b Show that the functions $y_1(x)=e^{-3x}$, $y_2(x)=\cos 2x$ are inealry independent.

13 a Find a particular solution of $y^{11}+3y^1+4y=3x+2$.

OR

b Transform the system of second-order equations $2x^{11}=-6x+2y$; $y^{11}=2x-2y+40\sin 3t$ into an equivalent first-order system.

14 a Prove that $L[af(t)+bg(t)]=aL[f(t)]+bL[g(t)]$ if a and b are constants.

OR

b Find $L^{-1}\left[\dfrac{s+1}{s(s+2)}\right]$

15 a Find the convolution of cost and sint.

OR

b Find $L\{f(t)\}$ if $f(t)=\begin{cases}\cos 2t & \text{if } 0\le t<2\pi\\ 0 & \text{if } t\ge 2\pi\end{cases}$

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a A lunar lander is falling freely toward the surface of the moon at a speed of 450 meters per second (m/s). Its retro rockets, when fired, provide a constant declaration of 2.5 meters per second (m/s^2) (the gravitational acceleration produced by the moon is assumed to be included in the given declaration). At what height above the lunar surface should the retrorockets be activated to ensure a "soft touch down" ($\upsilon=0$ at impact)?

OR

b Solve the differential equation $2xe^{2y}\dfrac{dy}{dx}=3x^4+e^{2y}$.

17 a Solve the initial value problem $y'''+3y''-10y'=0$ satisfying the initial conditions $y(0)=7$, $y^1(0)=0$, $y^{11}(0)=70$.

OR

b (i) Find a general solution of $y'''+y'-10y=0$.
(ii) Find a general solution of $y^{11}+b^2y=0$ $(b>0)$.

18 a Find a particular solution of $y^{11}-4y=2e^{2x}$.

OR

b Solve the two-dimensional system $x^1=-2y$, $y^1=x/2$

19 a Solve the system $2x^{11}=-6x+2y$; $y^{11}=2x-2y+40\sin 3t$ subject to the initial conditions $x(0)=x^1(0)=y(0)=y^1(0)=0$.

OR

b Find the inverse Laplace transform of $R(S)=\dfrac{s^2+1}{s^3-2s^2-8s}$.

20 a (i) Find $L\{t^2\sin kt\}$. (ii) Find $L^{-1}[\tan^{-1}(1/s)]$

OR

b Consider a mass on a spring with $m=k=1$ and $x(0)=x^1(0)=0$. At each of the instants $t=0,\pi,2\pi,\ldots,\eta\pi,\ldots$, the mass is struck a hammer blow with a unit impulse. Determine the resulting motion.