PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2019
(First Semester)

Branch – **MATHEMATICS**

CLASSICAL ALGEBRA & TRIGONOMETRY

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 Prove that the series $1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots$ is convergent.

2 State the Cauchy's condensation test.

3 State the Raabe's test.

4 Show that the series $\sum_{n=1}^{\infty}\frac{1}{n^3+n^4x^2}$ is uniformly convergent for all values of x.

5 Form a rational cubic equation which shall have for roots $1,3-\sqrt{-2}$.

6 Define reciprocal equation.

7 Find $\lim_{\theta\to 0}\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$.

8 Prove that $\cosh^2x+\sinh^2x=\cosh 2x$.

9 Find Log(iy).

10 Find the sum of the series $\operatorname{cosec}\theta+\operatorname{cosec}2\theta+\operatorname{cosec}2^2\theta+\ldots+\operatorname{cosec}2^{n-1}\theta$.

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a If $u_1+u_2+\ldots$ is convergent and has the sum S, then prove that $ku_1+ku_2+\ldots$ is convergent and has the sum kS.

OR

b Find whether the series in which $u_n=(n^3+1)^{1/3}-n$ is convergent or divergent.

12 a Examine the convergency of the series $\sum_1^{\infty}\left(\frac{n}{n+1}\right)^{1/2}x^n$.

OR

b Examine the convergence the series $\sum\frac{(n+1)(n+2)\ldots(n+n)}{n^n}$.

13 a Show that if a,b,c are real, the roots of $\frac{1}{x+a}+\frac{1}{x+b}+\frac{1}{x+c}=\frac{3}{x}$ are real.

OR

b If $\alpha,\beta,\gamma,\delta$ are the roots of the equation $x^4+px^3+qx^2+rx+s=0$, find (i) $\Sigma\alpha^2$ (ii)$\Sigma\alpha^2\beta\gamma$ (iii) $\Sigma\alpha^2\beta^2$ and (iv) $\Sigma\alpha^3\beta$.

14 a Express $\cos 8\theta$ interms of $\sin\theta$.

OR

b Separate into real and imaginary parts $\tan^{-1}(x+iy)$.

Cont...

15 a Show that $\log_i i = \frac{4n+1}{4m+1}$, where m and n are integers.

OR

b Sum the series $\sin^3\frac{\theta}{3} + 3\sin^3\frac{\theta}{3^2} + 3^2\sin^3\frac{\theta}{3^3} + \ldots$ to n terms.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 If $a_{n+2} = \sqrt{a_{n+1}a_n}$ and $a_n>0$, show that the sequence $\{a_{2n-1}\}$ and $\{a_{2n}\}$ are both monotonic, one increasing and the other decreasing and the sequence $\{a_n\}$ tends to $\left(a_1 a_2^{\,2}\right)^{1/3}$.

17 Test for convergency and divergency the series $1 + \frac{2x}{2!} + \frac{3^2x^2}{3!} + \frac{4^3x^3}{4!} + \frac{5^4x^4}{5!} + \ldots$

18 Solve the equation $6x^5-x^4-43x^3+43x^2+x-6=0$.

19 Expand $\sin^3\theta\cos^5\theta$ in a series of multiples of θ.

20 Find the sum to infinity of the series $\sin\alpha + c\ \sin(\alpha+\beta) + \frac{c^2}{2}\sin(\alpha+2\beta) + \ldots$. when $|c|<1$.

Z-Z-Z END