PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION DECEMBER 2019
(First Semester)

Branch – **MATHEMATICS**

CALCULUS

Time : Three Hours | Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 If $x^3+y^3=3axy$, find $\frac{dy}{dx}$.

2 Find the maximum value for the function $\frac{\log x}{x}$.

3 Write down the Cartesian formula for Radius of Curvature?

4 Define Evolute.

5 Show that $\int_0^{\pi} \cos^3 x dx = 0$.

6 Evaluate $\int xe^x dx$.

7 Evaluate $\int_0^1 \int_1^2 (x^2+y^2)dxdy$.

8 Find $\int_0^3 \int_0^2 \int_0^1 (x+y+z)dzdxdy$.

9 Prove that $\Gamma \frac{1}{2} = \sqrt{\pi}$

10 What is the relation connecting Beta and Gamma functions?

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Find the maxima and minima of the function $2x^3-3x^2-36x+10$.

OR

b Find the minimum value of $x^2+y^2+z^2$ subject to the condition $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$.

12 a Find the centre of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $\left(\frac{a}{4},\frac{a}{4}\right)$.

OR

b Find the radius of curvature at (x,y) for the curve $a^2y=x^3-a^3$.

13 a Evaluate $\int_0^1 \frac{\log(1+x)}{(1+x^2)} dx$.

OR

b Find the reduction formula for $\int \sin^n x dx$.

14 a Evaluate $\iiint_R (x-y+z)dx\ dy\ dz$ where R is given by $1 \le x \le 2; 2 \le y \le 3$, $1 \le z \le 3$.

OR

b Evaluate $\int\int (x^2+y^2)dxdy$ over the area bounded by the curves y=4x,

15 a Find $\int_0^{\pi/2} \sin^4\theta \cos^6\theta \, d\theta$.

OR

b Evaluate $\int_0^1 \frac{1}{\sqrt{1+x^3}} dx$.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Using Lagrange's method of multipliers, to find the minimum value of $x^2+y^2+z^2$ given that $ax+by+cz=p$.

17 Find the evolute of the curve $x^{2/3} + y^{2/3} = a^{2/3}$

18 Derive the reduction formula for $\int x^m (\log x)^n dx$, m and n are positive integers and hence evaluate $\int x^3 (\log x)^2 dx$.

19 Verify that $\iint_R (x^2 + y^2) dy dx = \iint_R (x^2 + y^2) dx dy$ where the region R is the triangle formed by the lines y=0, x=1 and y=x.

20 Evaluate $\int_0^1 x^m \left(\log \tfrac{1}{x}\right)^n dx$ and hence find the value of $\int_0^1 x^4 \left(\log \tfrac{1}{x}\right)^3 dx$.

Z-Z-Z END