18MAU04

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2019
(Second Semester)

Branch – **MATHEMATICS**

CALCULUS - II

Time: Three Hours — Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent when

(i) $p < 1$ (ii) $p \leq 1$
(iii) $p = 1$ (iv) $p > 1$

2 Find the sum 8 + 4 + 2 + 1 + …..

(i) $S_\infty = 8$ (ii) $S_\infty = 16$
(iii) $S_\infty = 24$ (iv) $S_\infty = 18$

3 The series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent then what is the condition of L in the root test?

(i) $L = 1$ (ii) $L > 1$
(iii) $L < 1$ (iv) $L \geq 1$

4 Find the radius of convergence of the series $\sum_{n=1}^{\infty} nx^{n-1}$

(i) $R = 1$ (ii) $R = 0$
(iii) $R = \infty$ (iv) $R = 2$

5 What is the necessary and sufficient condition for the integral $\int_c F.dr$ is to be independent for every closed path C in D?

(i) $\int_c F.dr = \infty$ (ii) $\int_c F.dr = 0$
(iii) $\int_c F.dr = 1$ (iv) $\int_c F.dr = -\infty$

6 If $F(x, y, z) = xz\bar{i} + xyz\bar{j} - y^2\bar{k}$, find divF,

(i) $z - xz$ (ii) $y + xz$
(iii) $z + xz$ (iv) $y - xz$

7 Label that Mobius strip surface has

(i) Only one side (ii) Two-sides only
(iii) Orientable surface (iv) None

8 Identify the expression of a stokes' theorem

(i) $\int_c F.dr = \iint_s \text{curl } F.ds$ (ii) $\int_c F.dr = \iint_s \text{div } F.ds$
(iii) $\int_c F.dr = \int_s \text{curl } F.ds$ (iv) $\int_c F.dr = \iint_s \text{div curl } F.ds$

9 If $x = \cos\theta + i\sin\theta$ then find $\cos\theta$

(i) $\cos\theta = \frac{1}{2}(x^n + \frac{1}{x^n})$ (ii) $\cos\theta = \frac{1}{2}(x - \frac{1}{x})$

(iii) $\cos\theta = \frac{1}{2}(x + \frac{1}{x})$ (iv) $\cos\theta = \frac{1}{2}(x^n - \frac{1}{x})$

10 What is the expansion of $\sin\theta$ in powers of θ?

(i) $\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - .....$ (ii) $\sin\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - .....$

(iii) $\sin\theta = \theta + \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - .....$ (iv) $\sin\theta = 1 + \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - .....$

SECTION - B (25 Marks)ss

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Determine the sum of the series $\sum_{n=1}^{\infty}\left(\frac{3}{n(n+1)} + \frac{1}{2^n}\right)$.

OR

b State and prove that comparison test of convergency.

12 a i) Write all the conditions of the root test.

ii) Test the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{2n+3}{3n+2}\right)^n$.

OR

b For what values of x does the series $\sum_{n=1}^{\infty}\frac{(x-3)^n}{n}$ converge?

13 a Evaluate $\int_c 2x\,ds$ where c consists of the arc c_1 of the parabola $y = x^2$ from (0, 0) to (1, 1) followed by the vertical line segment c_2 from (1, 1) to (1, 2).

OR

b Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

14 a Find the tangent plane to the surface with parametric equations $x = u^2$, $y = v^2$ and $z = u + 2v$ at the point (1, 1, 3).

OR

b Use stokes' theorem to compute the integral $\iint_s \text{curl}\, F.ds$ where $F(x, y, z) = xz\bar{i} + yz\bar{j} + xy\bar{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy = plane.

15 a If α, β, γ be the roots of the equation $x^3 + px^2 + qx + p = 0$, prove that $\tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma = n\lambda$ radians except when q = 1.

OR

b If $\tan A = \tan\alpha \tanh\beta$, $\tan B = \cot\alpha \tanh\beta$, state $\tan(A + B) = \sinh 2\beta \operatorname{cosec} 2\alpha$.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a i) Find the sum of the series $\sum_{n=0}^{\infty} x^n$ where $|x| < 1$.

ii) Find the sum of the series $\sum_{n=1}^{\infty}\left(\frac{3}{n(n+1)}+\frac{1}{2^n}\right)$.

OR

b Use the sum of the first 100 terms to approximate the sum of the series $\sum \frac{1}{(n^3+1)}$. Estimate the error involved in this approximation.

17 a Discover the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ correct to three decimal places.

OR

b Examine the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$.

18 a A wire takes the shape of the semicircle $x^2+y^2=1$, $y \geq 0$ and is thicker near its base than near the top. Discover the center of mass of the wire if the linear density at any point is proportional to its distance from the line $y = 1$.

OR

b If $F(x, y) = (-y\bar{i}+x\bar{j}) / (x^2 + y^2)$ then point out the integral $\int_c F.dr$ as 2π for every positively oriented simple closed path that encloses the origin.

19 a Evaluate $\iint_s z\,ds$, where S is the surface whose sides S, are given by the cylinder $x^2 + y^2 = 1$, whose bottom S_2 is the disk $x^2 + y^2 \leq 1$ in the plane $z = 0$, and whose top S_3 is the part of the plane $Z = 1 + x$ that lies above S_2.

OR

b Apply divergence theorem to evaluate $\iint_s F.ds$ where $F(x, y, z) = xy\bar{i}+(y^2+e^{xz^2})\bar{j}+\sin(xy)\bar{k}$ and S is the surface of the region E bounded by the parabolic cylinder $z = 1 - x^2$ and the planes $z = 0$, $y = 0$ and $y + z = 2$.

20 a Point out the equation

$$\frac{ah}{\cos\theta} - \frac{bk}{\sin\theta} = a^2 - b^2$$

Has four roots and that the sum of the four values of θ which satisfy it is equal to an odd multiple of π radians.

OR

b Classify the real and imaginary parts of $\tan^{-1}(x+iy)$.

Z-Z-Z END