PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION DECEMBER 2019
(First Semester)

Branch – **MATHEMATICS**

CALCULUS – I

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 1 = 10)

1 The curvature of the parabola $y=x^2$ at the point (0,0) is _____.
(i) 0 (ii) 1 (iii) 2 (iv) 3

2 If $\vec{r}(t) = -\sin t\,\vec{i} + \cos t\,\vec{j} + \vec{k}$ then $|\vec{r}(t)| =$ _____.
(i) 2 (ii) $\sqrt{2}$ (iii) 3 (iv) $\sqrt{3}$

3 $\lim_{(x,y)\to(1,2)} (5x^3 - x^2y^2) =$ _____.
(i) 5 (ii) 4 (iii) 3 (iv) 1

4 If $f(x,y)=4-x^2-2y^2$, then $f_x(1,1)=$_____.
(i) -2 (ii) 2 (iii) -4 (iv) 4

5 If $z=x^2y+3xy^4$, where x=sin2t and y=cost, then $\frac{dz}{dt}$ when t=0 is ____.
(i) 4 (ii) 5 (iii) 6 (iv) 7

6 The gradient of $f(x,y)=\sin x+e^{xy}$ at (0,1) is _____.
(i) <2,0> (ii) <2,1> (iii) <0,2> (iv) <1,2>

7 $\int_1^4 \int_0^2 (6x^2y - 2x)\,dy\,dx =$ _____.
(i) 2 (ii) 22 (iii) 222 (iv) 232

8 $\int_{b/\pi}^{b} \int_0^{\pi/2} r\,d\theta\,dr =$ ____.
(i) $\frac{3}{16}\pi b^2$ (ii) $\frac{3}{16}b^2$ (iii) $3\pi b^2$ (iv) $3\pi b$

9 The cylindrical coordinates of the point with rectangular coordinates $\left(2,\frac{2\pi}{3},1\right)$ is ___.
(i) $\left(2,\frac{2\pi}{3},1\right)$ (ii) $(-1,\sqrt{3},2)$ (iii) $(-1,\sqrt{3},1)$ (iv) $(-1,\sqrt{3},-1)$

10 If u=x+y and v=x-y, then $\frac{\partial(u,v)}{\partial(x,y)} =$ _____.
(i) -2 (ii) -½ (iii) ½ (iv) -1

SECTION - B (25 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 x 5 = 25)

11 a Find a vector equation and parametric equations for the line segment that joins the point P(1,3,-2) to the point Q(2,-1,3).

OR

b Show that the curvature of a circle of radius 'a' is $\frac{1}{a}$.

12 a Find the domain of the following function and evaluate f(3,2): $f(x,y)=\frac{\sqrt{x+y+1}}{x-1}$.

OR

13 a If $z=e^x \sin y$, where $x=st^2$ and $y=s^2t$, then find $\frac{\partial z}{\partial s}$.

OR

b Find the shortest distance from the point (1,0,-2 to the plane $x+2y+z=4$.

14 a Evaluate $\int_0^3 \int_1^2 x^2 y\,dy\,dx$.

OR

b Evaluate $\int_{\pi/4}^{3\pi/4} \int_1^2 r\,dr\,d\theta$.

15 a Evaluate $\iiint_E z\,dv$, where E is the solid tetrahedron bounded by the four planes $x=0$, $y=0$, $z=0$ and $x+y+z=1$.

OR

b The point $\left(2, \frac{\pi}{4}, \frac{\pi}{3}\right)$ is given in spherical coordinates. Find its rectangular coordinates.

SECTION -C (40 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 8 = 40)

16 a Find the unit tangent vector at the point where t=0 for $\vec{r}(t) = (1+t^3)\vec{i} + te^{-t}\vec{j} + \sin 2t\vec{k}$.

OR

b Find the unit normal and binormal vectors for the circular helix $\vec{r}(t) = -\cos\vec{i} + \sin t\vec{j} + t\vec{k}$.

17 a If $f(x,y) = \sin\left(\frac{x}{1+y}\right)$, then calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

OR

b Find the tangent plane to the elliptic paraboloid $z=2x^2+y^2$ at the point (1,1,3).

18 a Find y' if $x^3+y^3=6xy$.

OR

b Find the local maximum and minimum values and saddle points of $f(x,y)=x^4+y^4-4xy+1$.

19 a Evaluate $\iint_D (x+2y)\,dA$, where D is the region bounded by the parabolas $y=2x^2$ and $y=1+x^2$.

OR

b Find the volume of the solid bounded by the plane $z=0$ and the paraboloid $z=1-x^2-y^2$.

20 a Evaluate the triple integral $\iiint_B xyz^2\,dv$, where B is the rectangular box given by $B=\{(x,y,z)/0\le x\le 1, -1\le y\le 2, 0\le z\le 3\}$.

OR

b Evaluate $\iiint_B e^{\left(x^2+y^2+z^2\right)^{3/2}}\,dv$, where B is the unit ball: $B=\{(x,y,z)|x^2+y^2+z^2\le 1\}$.